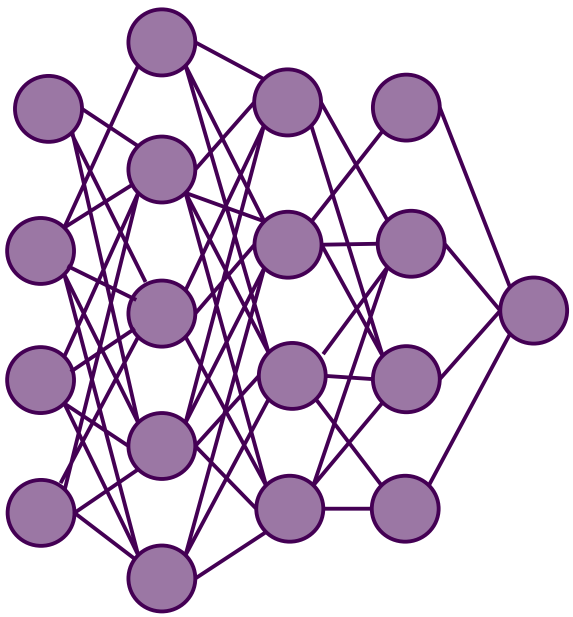
Variational wave-functions and neural networks

Agnes Valenti



Based on:

Lecture on NQS at ICTP 2024 (smr 3928) by Filippo Vicentini

Review article: M Medvidovic, J Robledo Moreno, arXiv:2402.11014 (2024)

Book: F Becca, “Quantum Monte Carlo approaches for correlated systems”

Part I
Machine learning for quantum many-body simulations

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Quantum many-body problem(s)

Schroedinger equation $\mathcal{H}|\Psi\rangle = \mathrm{i}\hbar\partial_t|\Psi\rangle$

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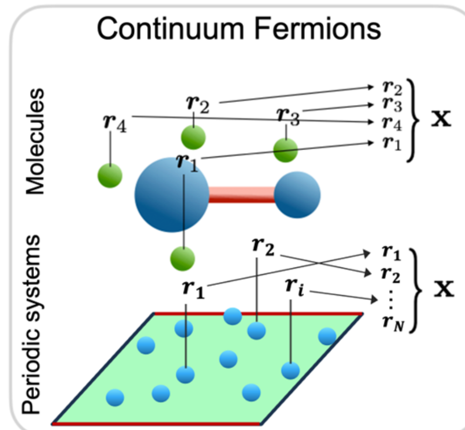

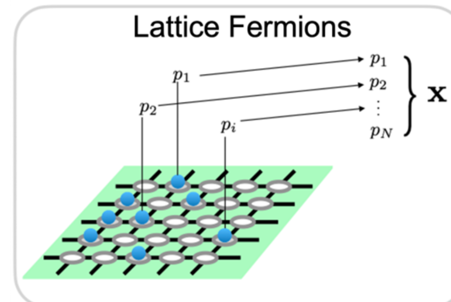

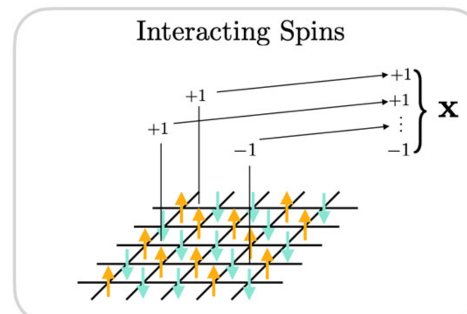

[M Medvidovic, J Robledo Moreno, arXiv:2402.11014 (2024)]

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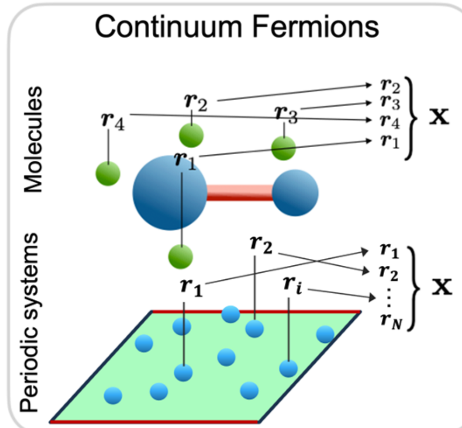
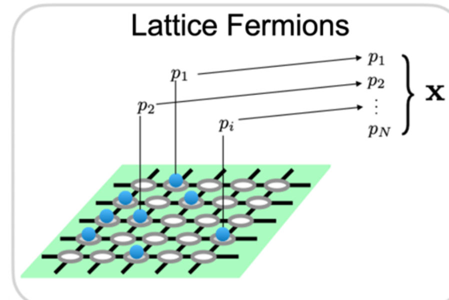
Quantum many-body problem(s)

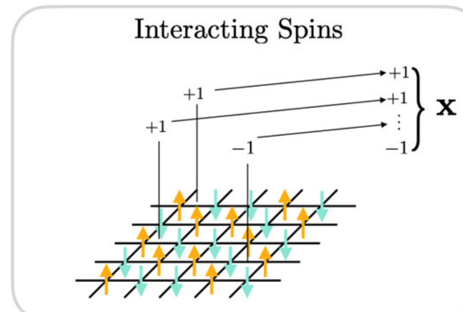
Schroedinger equation $\mathcal{H}|\Psi\rangle = i\hbar\partial_t|\Psi\rangle$

- Ground states $\mathcal{H}|\Psi\rangle = E|\Psi\rangle$
- Time evolution $|\Psi_0\rangle \rightarrow |\Psi(t)\rangle$
- Finite temperature $[\mathcal{H}, \rho] = i\hbar\partial_t\rho$
- Open systems $\mathcal{L}(\rho) = \partial_t\rho$

⋮







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$$|\Psi\rangle = \sum_{S} \Psi(S)|S\rangle$$

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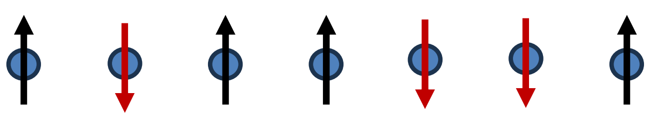
$S = s_1, \dots s_N$



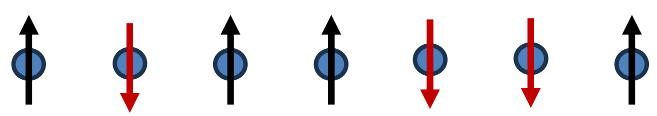
Basis state: Spin-configuration

Quantum many-body problem

Find the ground state $\mathcal{H}|\Psi\rangle = \mathrm{E}|\Psi\rangle$

$$|\Psi\rangle = \sum_{S} \Psi(S)|S\rangle = \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \vdots \\ \Psi_{2^N} \end{bmatrix}$$

$S = s_1, \ldots s_N$



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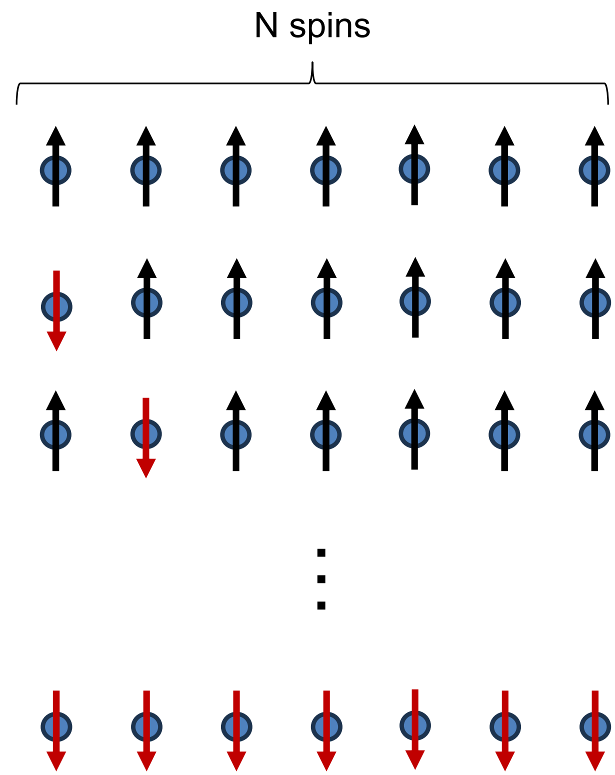

⇨ Hilbert space dimension 2^N

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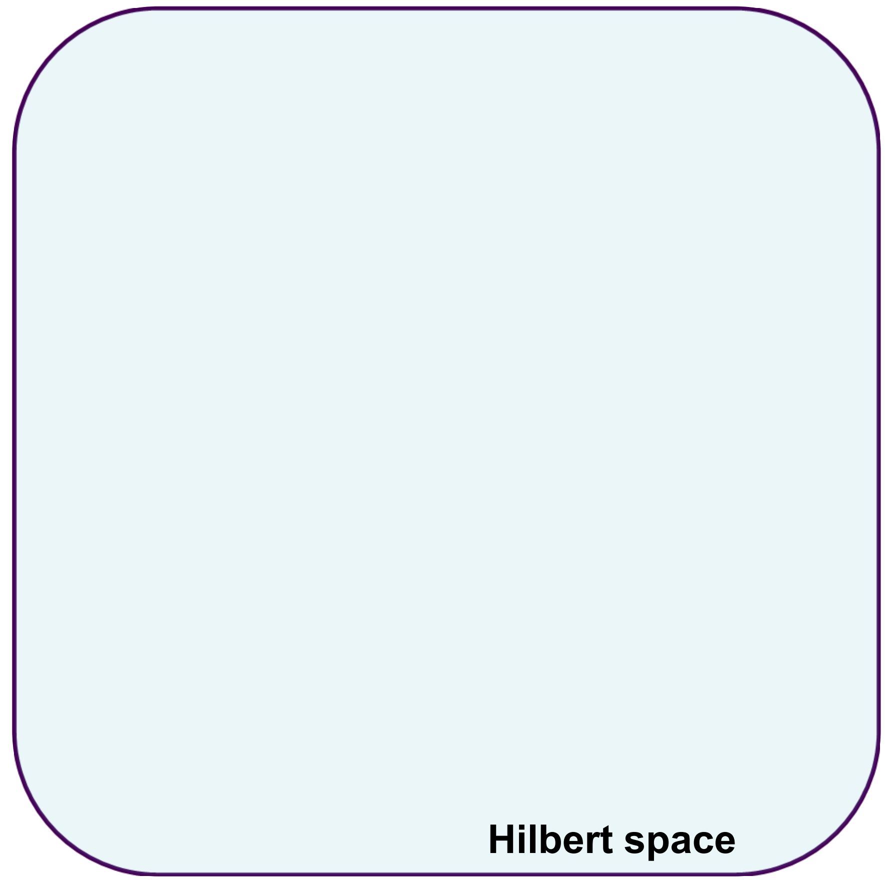
Variational wave-functions

Find the ground state $\mathcal{H}|\Psi\rangle = \mathrm{E}|\Psi\rangle$



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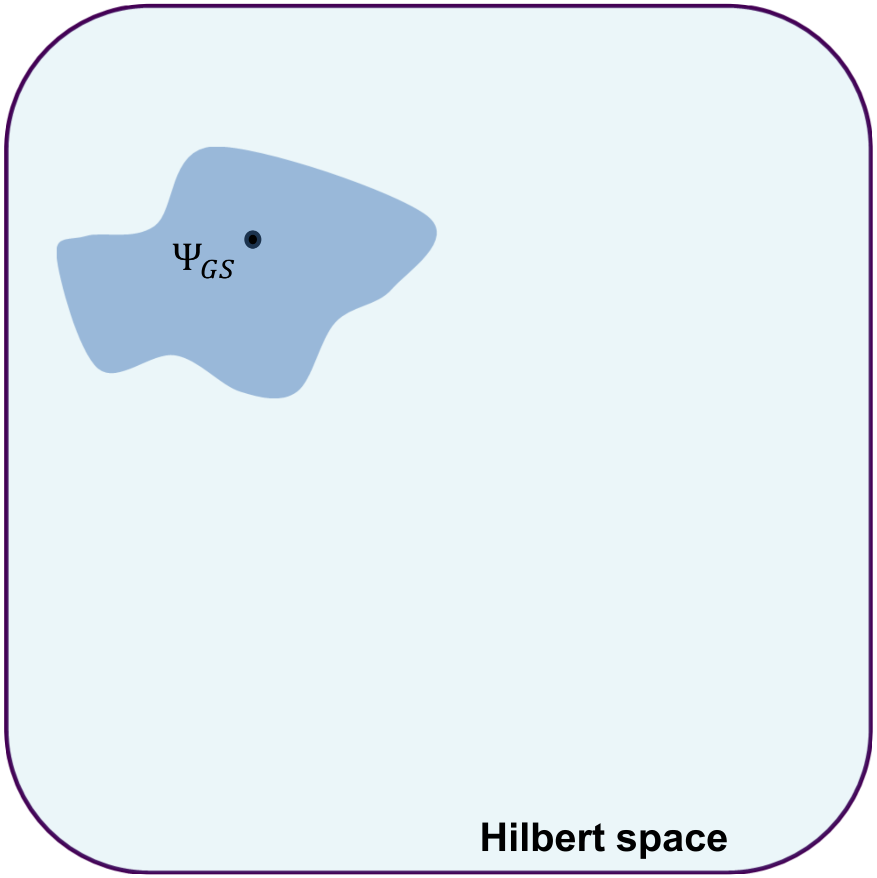

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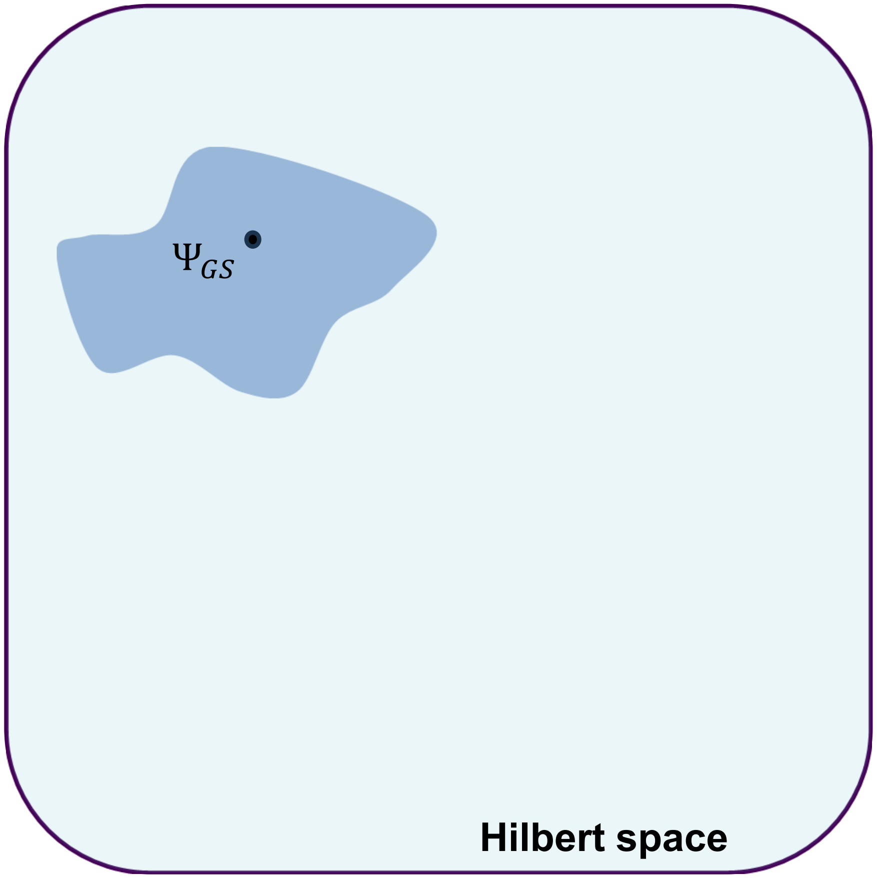

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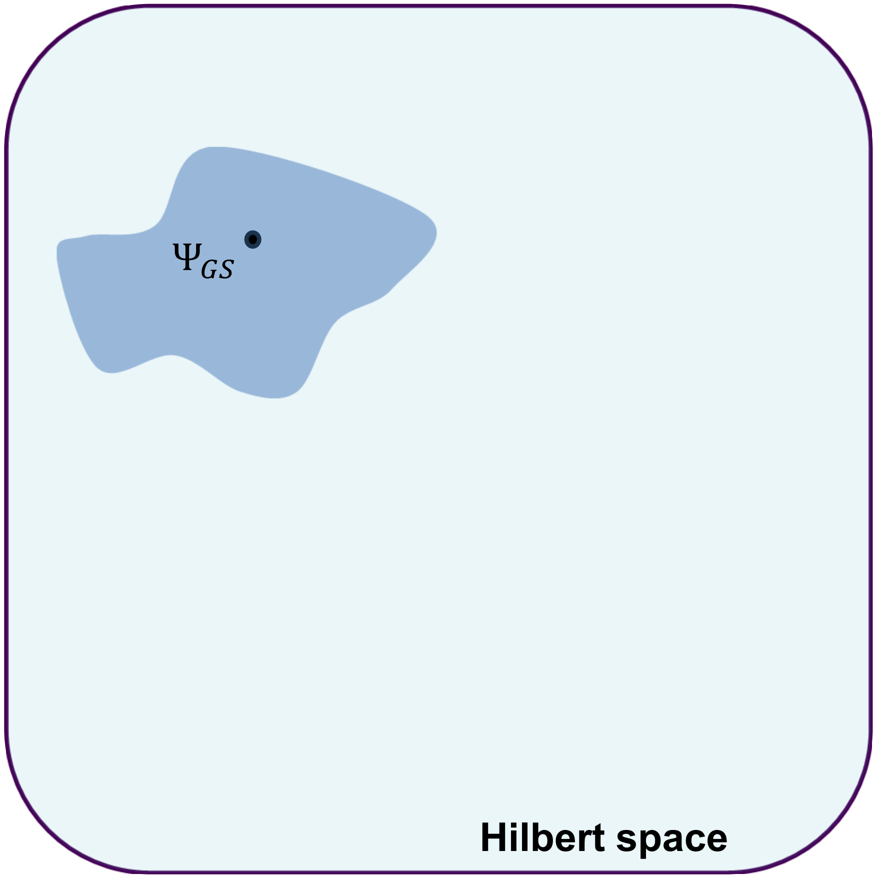

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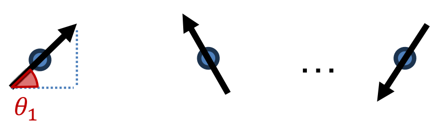
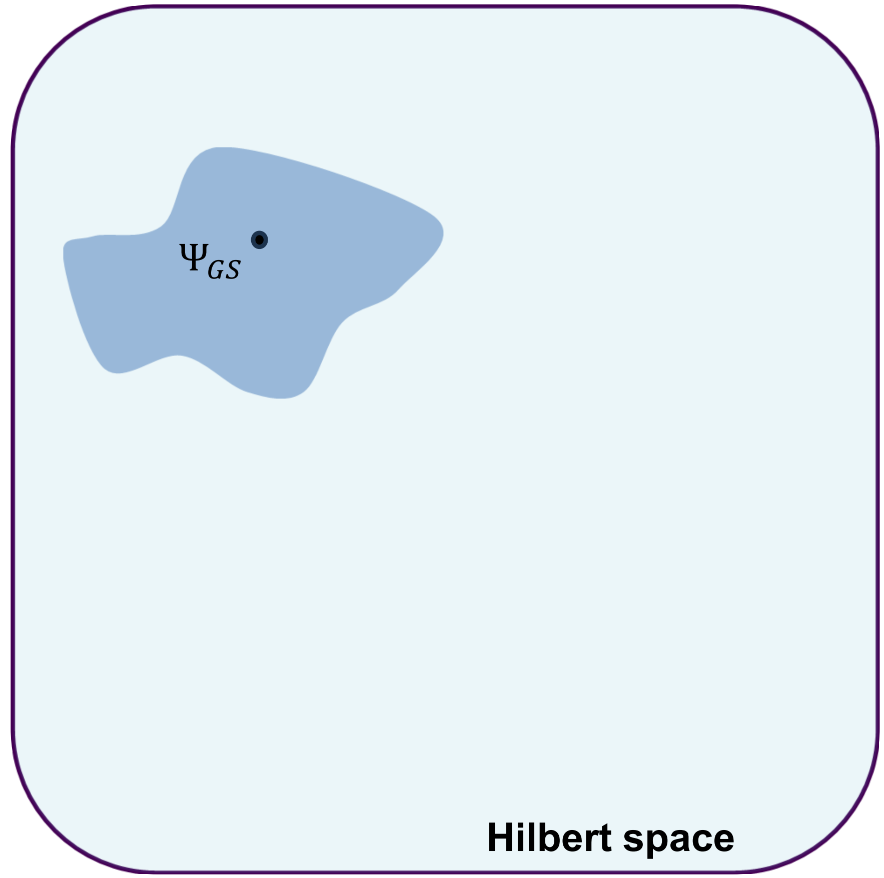
Examples:

1) Product (mean-field) ansatz

$$\Psi_{\boldsymbol{\theta}}(s_1, \dots s_N) = \Psi_{\theta_1}(s_1) \cdot \Psi_{\theta_2}(s_2) \cdot \dots \cdot \Psi_{\theta_N}(s_N)$$





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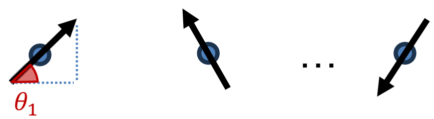
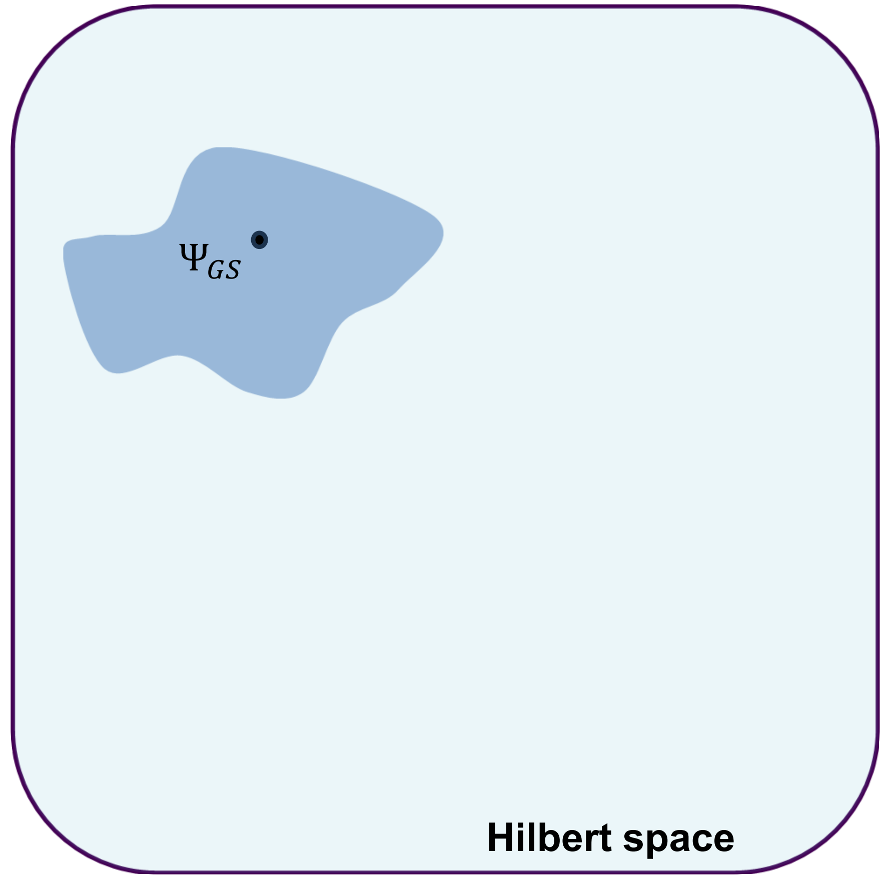


E.g. can not represent $|\Psi\rangle_{GHZ} = |0\rangle^{\otimes N} + |1\rangle^{\otimes N}$

(product state is not entangled)



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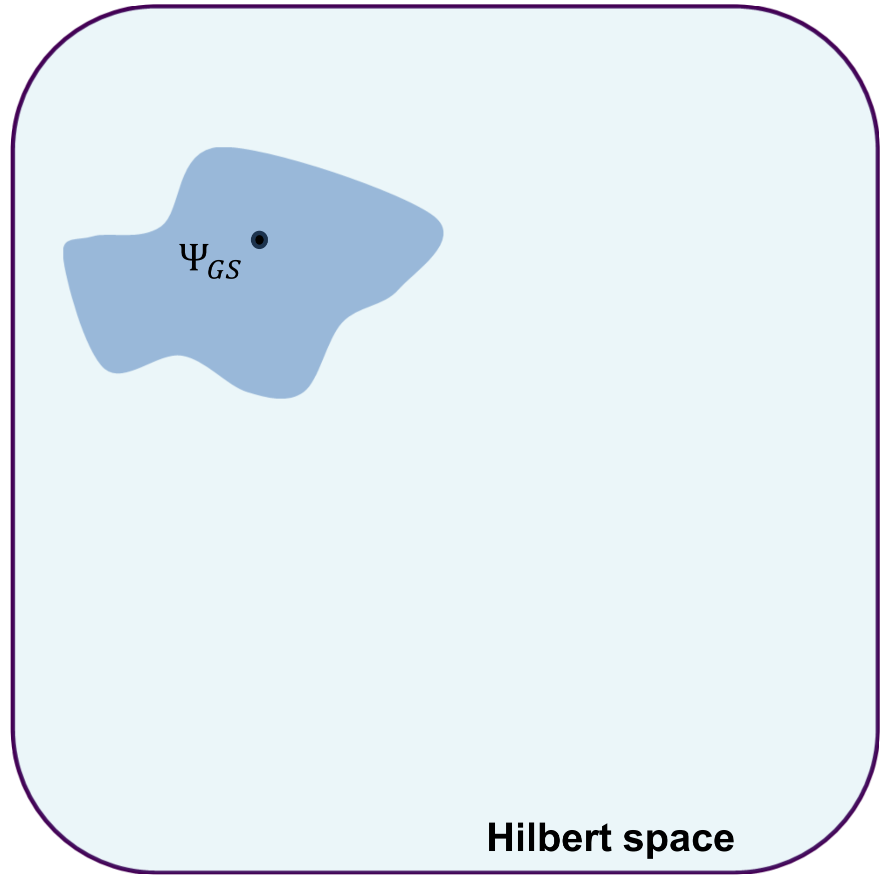
2) Matrix product states

$$\Psi_{\boldsymbol{\theta}}(s_1, \dots s_N) = \mathrm{Tr}[\boldsymbol{A}_1^{s_1} \boldsymbol{A}_2^{s_2} \dots \boldsymbol{A}_N^{s_N}]$$

Area-law entanglement



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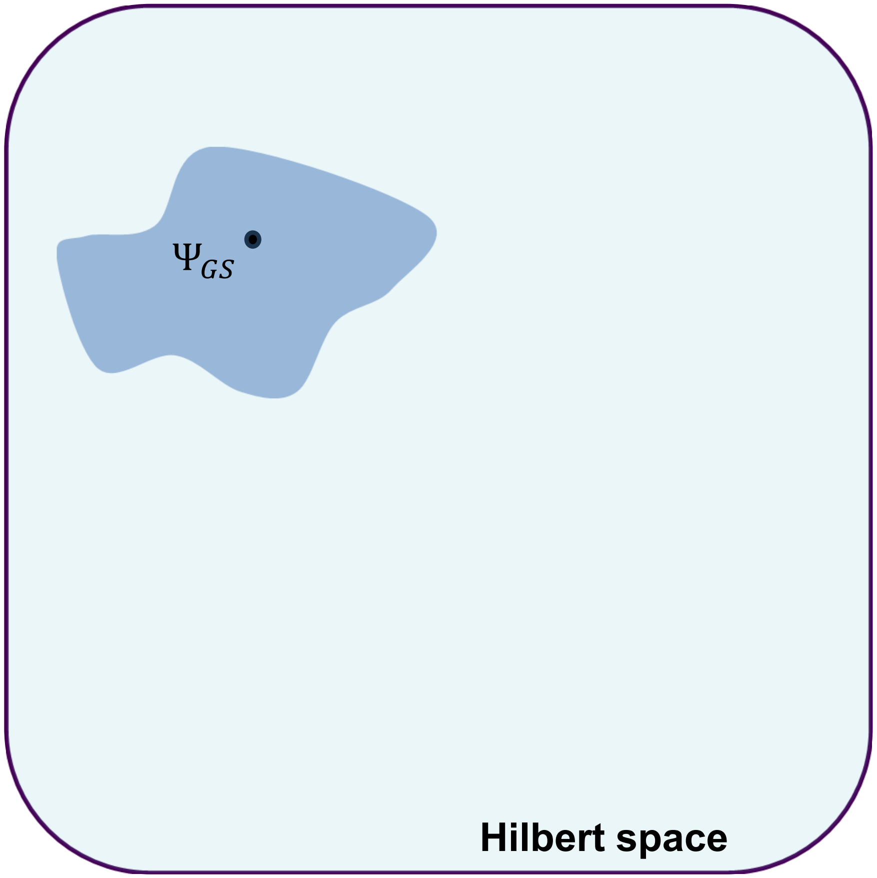
3) Neural-network quantum states

$$\Psi_{\boldsymbol{\theta}}(s_1, \dots s_N) = \mathrm{NN}_{\theta}(s_1, \dots s_N)$$



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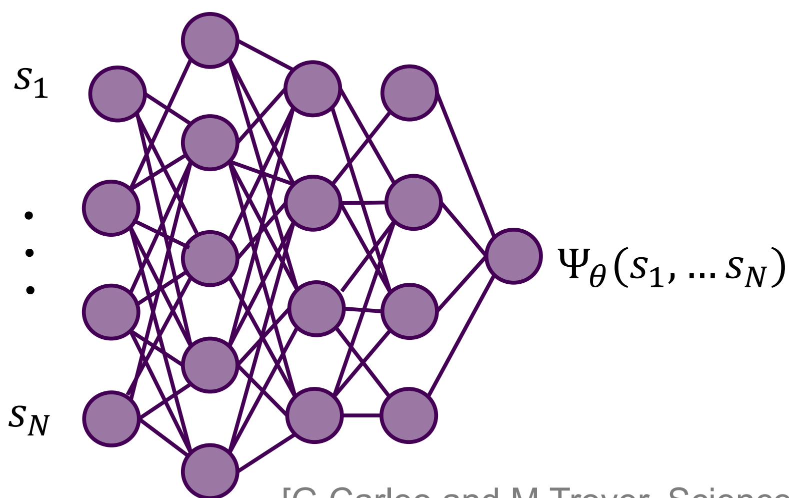
Neural-network quantum states

$$|\Psi\rangle = \sum_{s_1,\dots s_N} \underbrace{\Psi_\theta(s_1,\dots s_N)}_{\text{Neural-net output}} |s_1,\dots s_N\rangle$$



[G Carleo and M Troyer, Science (2017)]

[J Carrasquilla and R Melko, Nature Physics 13 (2017) - SI]

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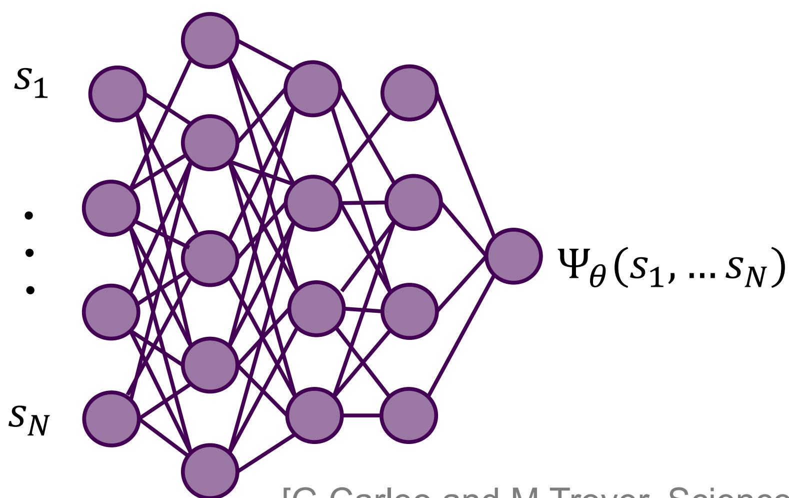

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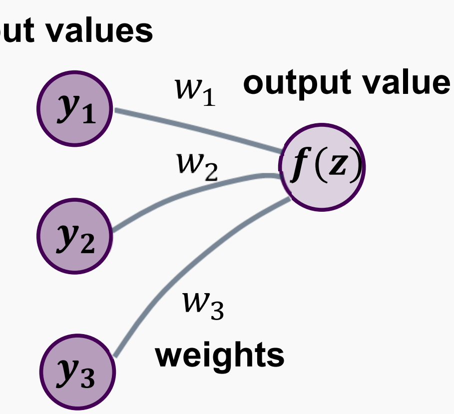
What is a neural network?

Output of a neuron = nonlinear function of weighted sum of inputs



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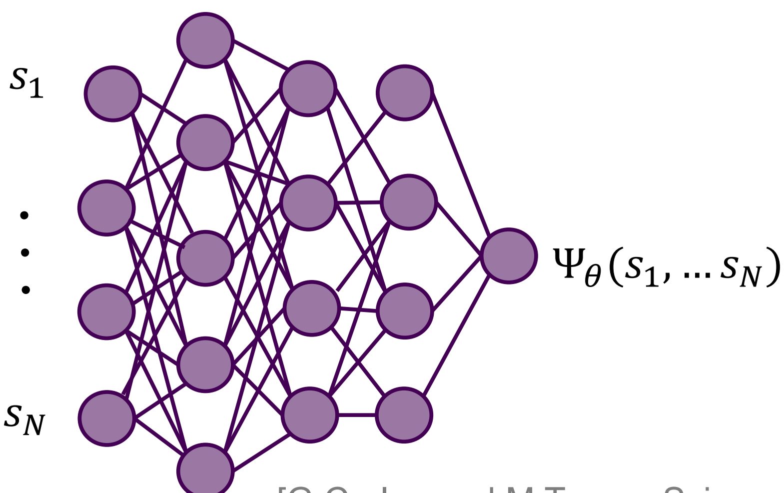

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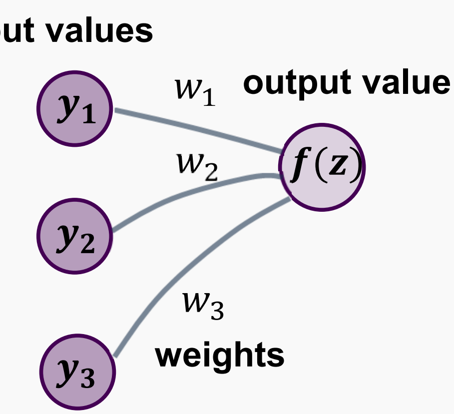

weighted sum $z = \sum_j w_j y_j + b$

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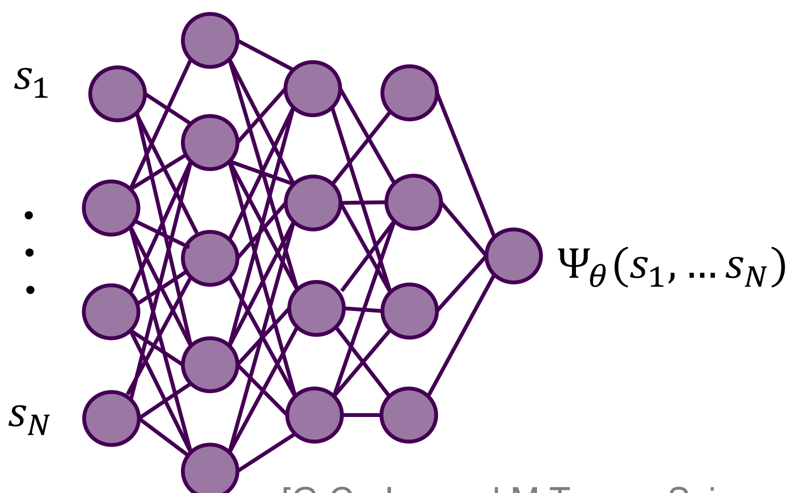

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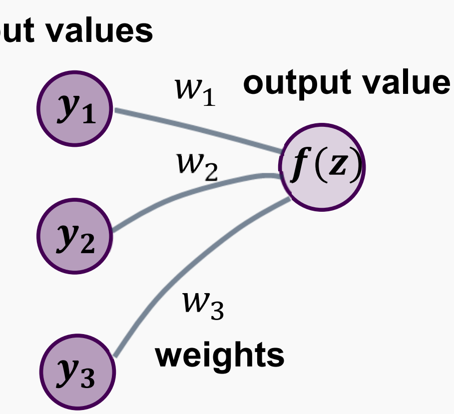
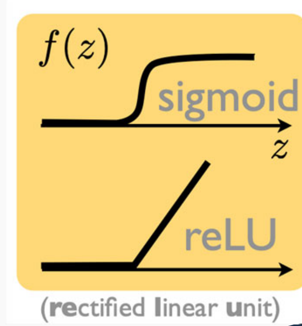


weighted sum $\quad z = \sum_j w_j y_j + b$

nonlinear "activation function"



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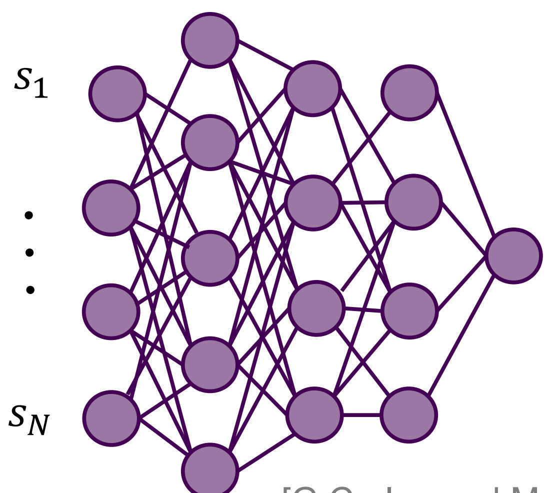

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Universal approximation theorem

single-layer neural network:

Width $\rightarrow \infty$ ⇨ $||f^* - f_W|| \sim \text{width}^{-1}$

[Cybenko, MCSS 2, 303 (1989)]
[Leshno et al., NN 6 861 (1993)]

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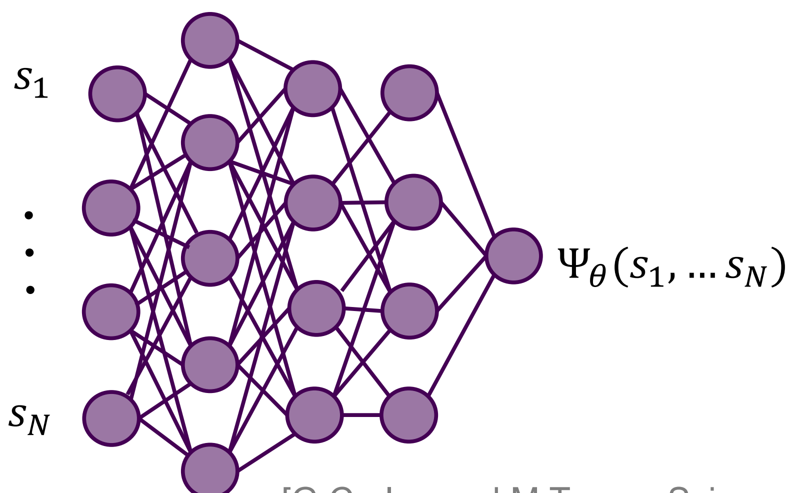

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Deep neural networks: $||f^* - f_W|| \sim \exp[-\text{depth}]$?

Proven in some exotic cases

[Z. Lu et al, NIPS 30, 6231 (2017)]

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Neural-network quantum states

$$|\Psi\rangle = \sum_{s_1,\dots s_N} \Psi_\theta(s_1, \dots s_N)|s_1, \dots s_N\rangle$$

Representability

- Always valid quantum state (unnormalized)

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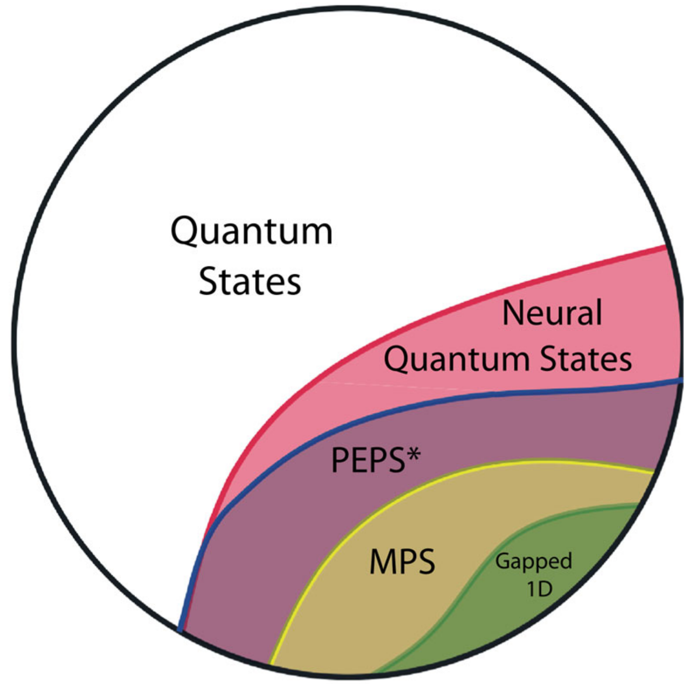
- Encode MPS and approximate PEPS with poly resources (BUT optimization harder + no tensor contraction) [Sharir, 2022]



[Sharir et al., PRB 106 (2022)]

[Carleo and Troyer, Science 355 (2017)]

[Carleo et al, MRP RMP 91(2019)]

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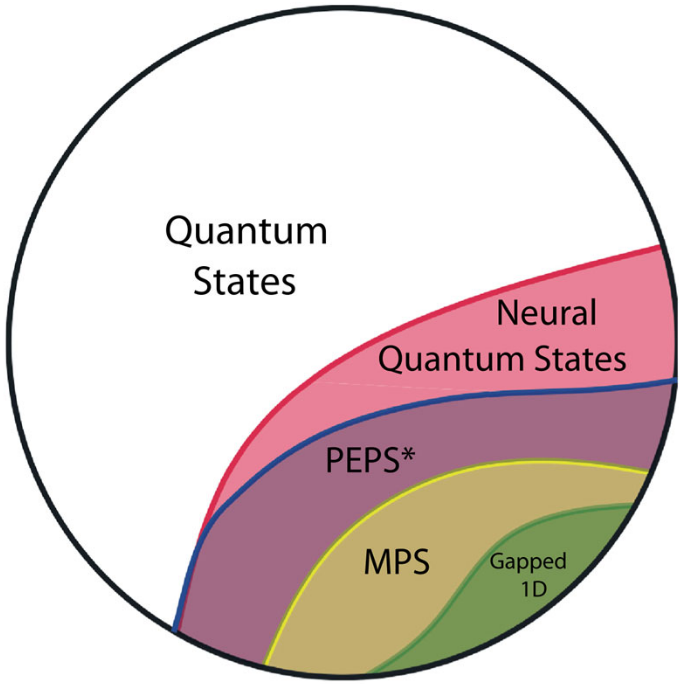
- Encode MPS and approximate PEPS with poly resources (BUT optimization harder + no tensor contraction) [Sharir, 2022]
- Volume-law in principle possible



[Sharir et al., PRB 106 (2022)]

[Carleo and Troyer, Science 355 (2017)]

[Carleo et al, MRP RMP 91(2019)]

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Part II
Variational Monte Carlo

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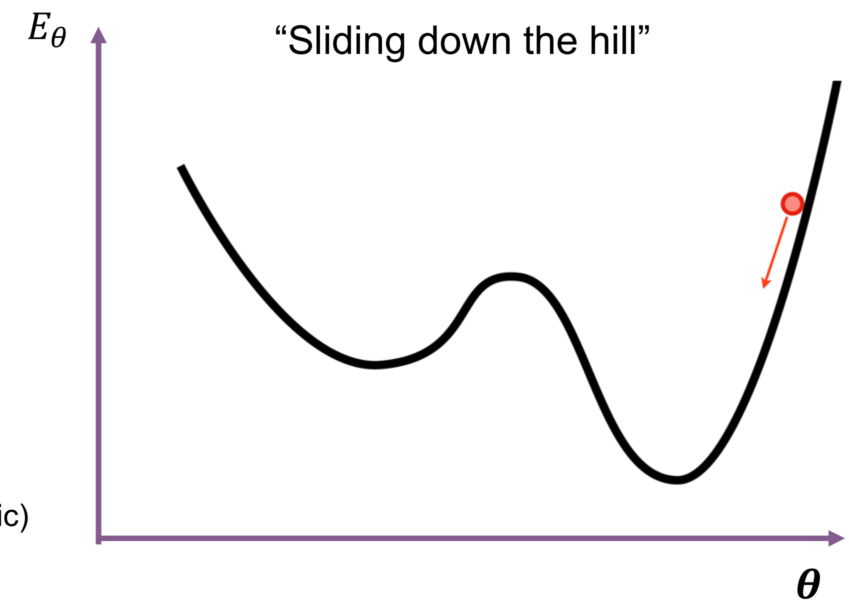
Stochastic optimization: ML perspective

Variational principle $E_\theta := \frac{\langle \Psi_\theta | H | \Psi_\theta \rangle}{\langle \Psi_\theta | \Psi_\theta \rangle} \geq E_G$

(Stochastic) Gradient descent $\theta_{new} = \theta - \eta \nabla_\theta \mathrm{E}_\theta$

with $\nabla_{\theta_k} E_\theta = \nabla_{\theta_k} \frac{\langle \Psi_\theta | H | \Psi_\theta \rangle}{\langle \Psi_\theta | \Psi_\theta \rangle} = 2\mathrm{Re}\{\underbrace{\langle O_k^\dagger H \rangle - \langle O_k^\dagger \rangle \langle H \rangle}_{\text{Estimated via sampling (stochastic)}}\}$

$O_k(S) = \frac{\partial \ln \Psi_\theta}{\partial \theta_k}(S)$



[Park and Kastoryano, PRR 2(2), 2020]

Not stable for NQS: Tends to oscillate between deep local minima

Improve by taking into account Riemannian metric

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Stochastic optimization: ML perspective

Variational principle $$E_\theta := \frac{\langle \Psi_\theta | H | \Psi_\theta \rangle}{\langle \Psi_\theta | \Psi_\theta \rangle} \geq E_G$$

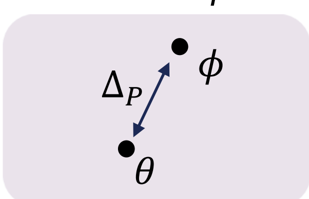
Natural gradient descent

$$\theta_{new} = \theta - \eta g^{-1} \nabla_\theta \mathrm{E}_\theta$$

Infinitesimal distance

$$||d\theta||^2 = \sum_{\alpha\beta} \underbrace{g_{\alpha\beta}(\theta)}_{\text{Metric tensor}} d\theta_\alpha d\theta_\beta \quad \Rightarrow \quad \nabla_\theta \rightarrow \tilde{\nabla}_\theta = g^{-1} \nabla_\theta$$

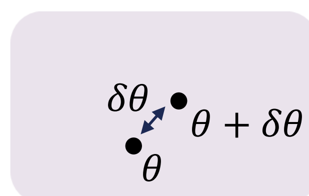
Parameter space



Euclidean distance

$$\Delta_P = |\vec{\theta} - \vec{\phi}|$$

Parameter space



$$ds^2 = \sum_\alpha d\theta_\alpha^2 = \sum_{\alpha,\beta} \boldsymbol{\delta_{\alpha\beta}} d\theta_\alpha d\theta_\beta$$

→"vanilla" gradient descent!

Stochastic optimization: ML perspective

Variational principle $E_\theta \coloneqq \frac{\langle \Psi_\theta | H | \Psi_\theta \rangle}{\langle \Psi_\theta | \Psi_\theta \rangle} \geq E_G$

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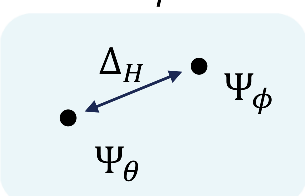
Hilbert space

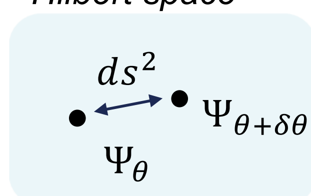


Fubini-Study distance

$$\Delta_H = \arccos |F_{\theta\phi}| ,$$

$$F_{\theta\phi} = \langle \Psi_\theta | \Psi_\phi \rangle$$

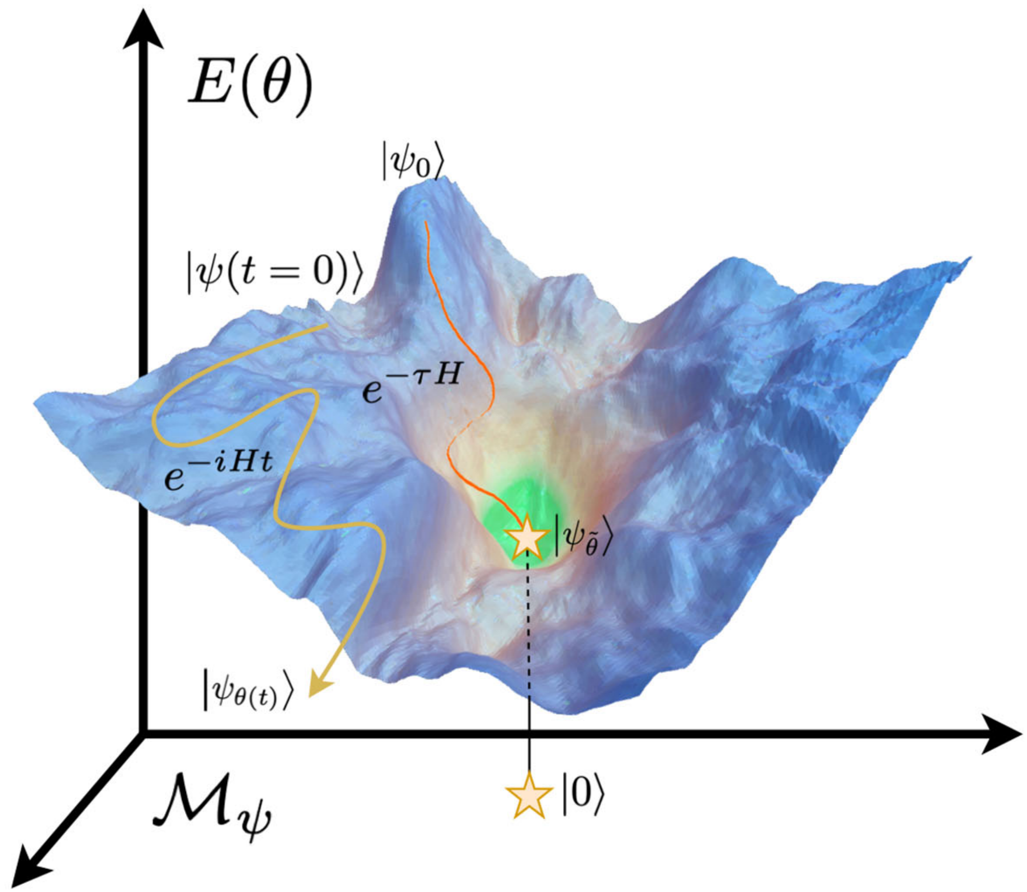
Hilbert space



$$ds^2 = \sum_{\alpha,\beta} \boldsymbol{S_{\alpha\beta}} d\theta_\alpha d\theta_\beta$$

$$S_{\alpha\beta} = \left\langle \frac{\partial \psi_\theta}{\partial \theta_\alpha} \middle| \frac{\partial \psi_\theta}{\partial \theta_\alpha} \right\rangle - \langle \frac{\partial \psi_\theta}{\theta_\alpha} | \psi_\theta \rangle \langle \psi_\theta | \frac{\partial \psi_\theta}{\partial \theta_\beta} \rangle$$

→**"Stochastic Reconfiguration"**


$E(\theta)$
$|\psi_0\rangle$
$|\psi(t=0)\rangle$
$e^{-\tau H}$
e^{-iHt}
$|\psi_{\tilde{\theta}}\rangle$
$|\psi_{\theta(t)}\rangle$
$|0\rangle$
$\mathcal{M}_\psi$

Symmetries

Set of symmetry operations $SG = \{g\}$:

$$\Psi\big(g(\boldsymbol{x})\big) = \chi_g^i \Psi(\boldsymbol{x})$$

↑ character

Symmetry sector i

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Direct symmetrization:

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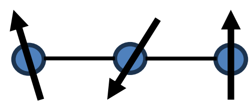
Example:

3 spins + PBC



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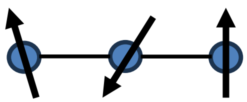

Translational symmetry:

	$\mathbf{1}$	$\boldsymbol{T}$	$\boldsymbol{T}^2$
χ^0	1	1	1
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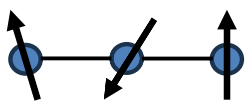

Symmetry sector 0:

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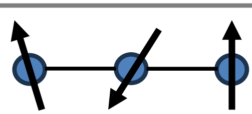

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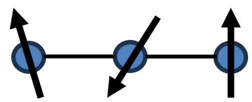

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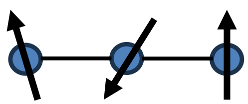

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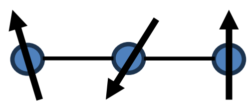

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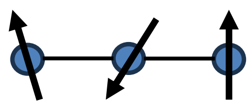

Symmetry sector 1:

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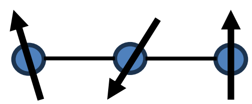

Symmetry sector 1:

$$\Psi_\theta^1(x) = \widetilde{\Psi}(x) + \omega\widetilde{\Psi}(T^{-1}x) + \omega^2\widetilde{\Psi}(T^{-2}x)$$

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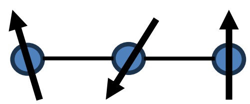

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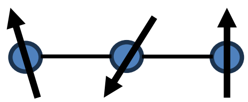

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Active area of research

[T Vieijra et al., PRL 124 (2020)]

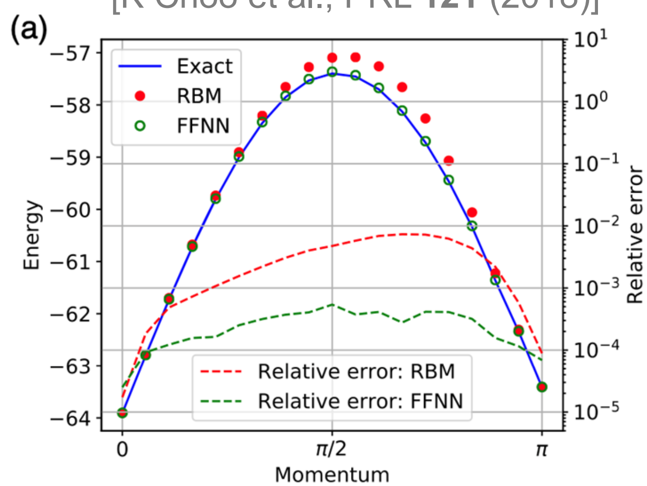
[T Vieijra and J Nys, PRB 104 (2021)]

[C Roth and A MacDonald, arXiv:2104 (2021)]

[M Reh et al., *PRB* 107 (2023)]

⋮

[K Choo et al., PRL **121** (2018)]

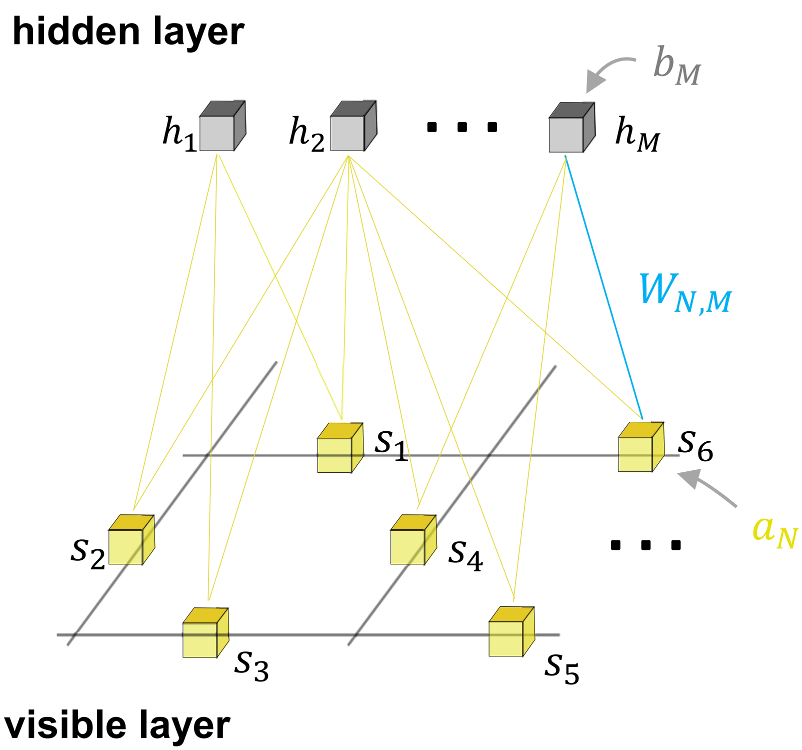


AFM Heisenberg $H = \sum_{\langle i,j \rangle} S_i \cdot S_j$

Part III
Applications: Architectures and benchmarks

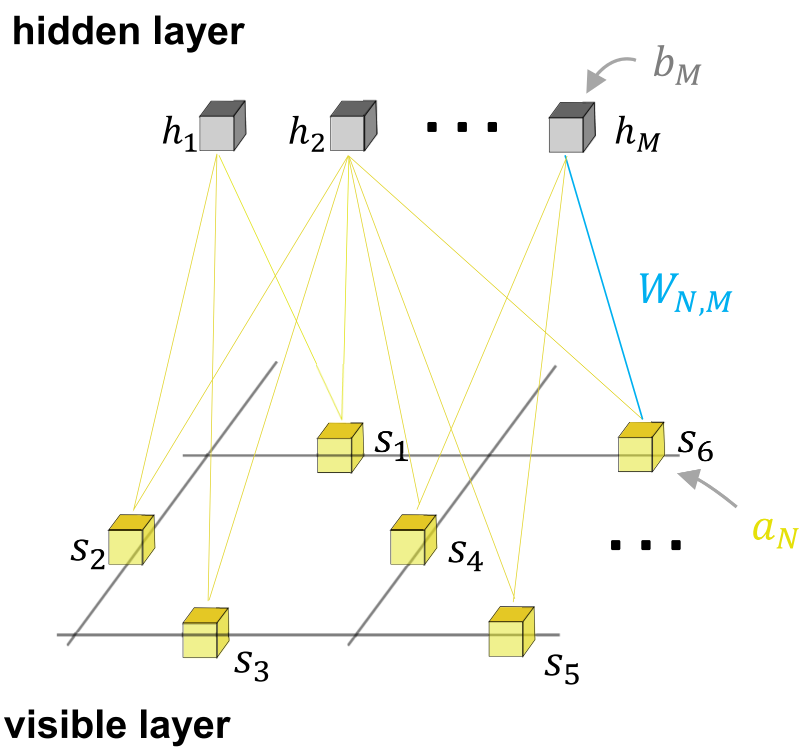
Feed-forward architectures

Feed-forward architectures: Restricted Boltzmann Machine (RBM)



[G Carleo and M Troyer, Science (2017)]

Feed-forward architectures: Restricted Boltzmann Machine (RBM)



- Variational state: $\Psi_\theta(S) = \sum_h \exp(E_{RBM})$,

  $\mathrm{E_{RBM}} = \sum_k a_k s_k + \sum_j b_j h_j + \sum_{k,j} W_{k,j} s_k h_j$

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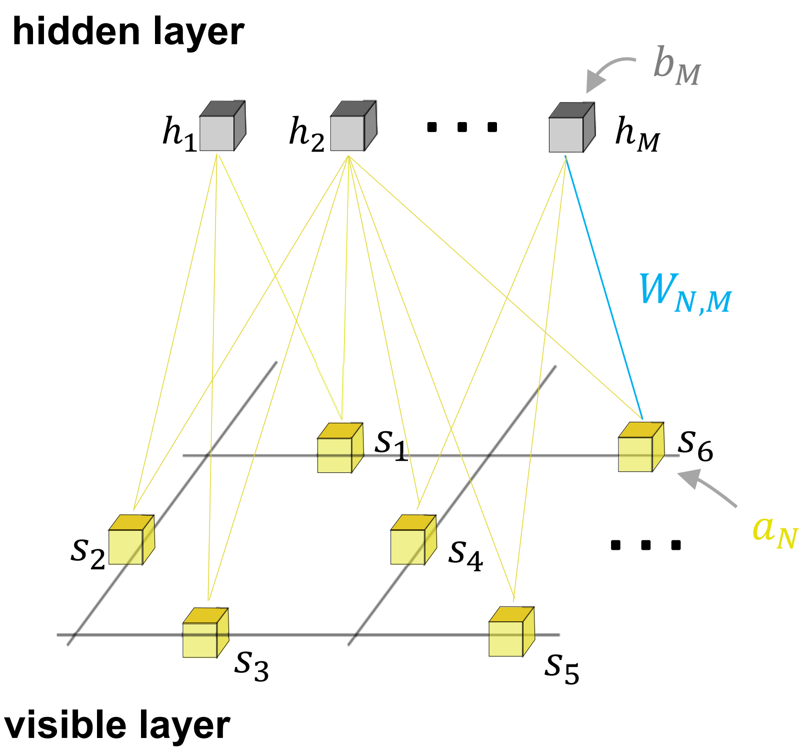

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- "factorize":

$$\Psi_\theta(S) = e^{\sum_k a_k s_k} \prod_j 2\cosh\left(\sum_{k,j} W_{k,j} s_k h_j + b_j\right)$$

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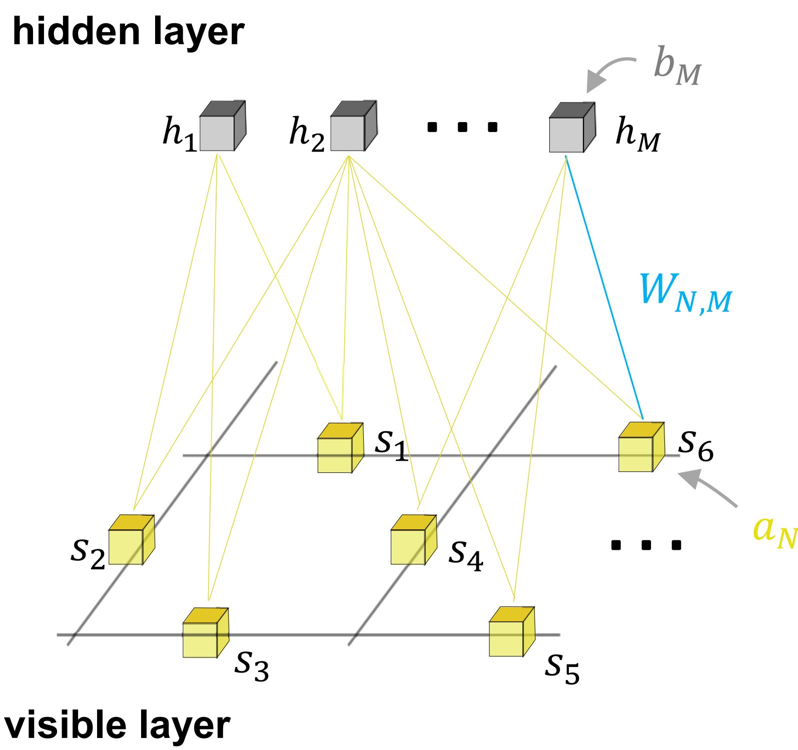
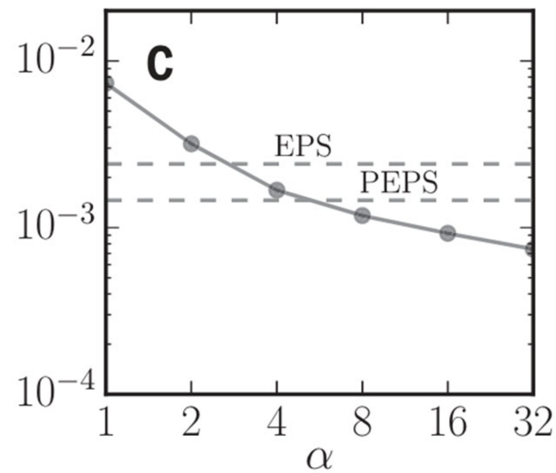
Feed-forward architectures: Restricted Boltzmann Machine (RBM)





AF Heisenberg

$$H = \sum_{\langle i,j \rangle} S_i \cdot S_j$$

10 × 10 lattice

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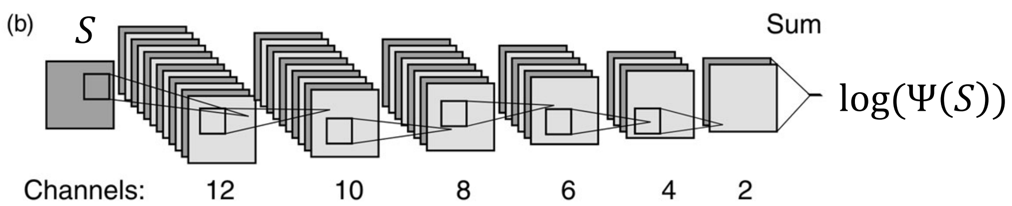
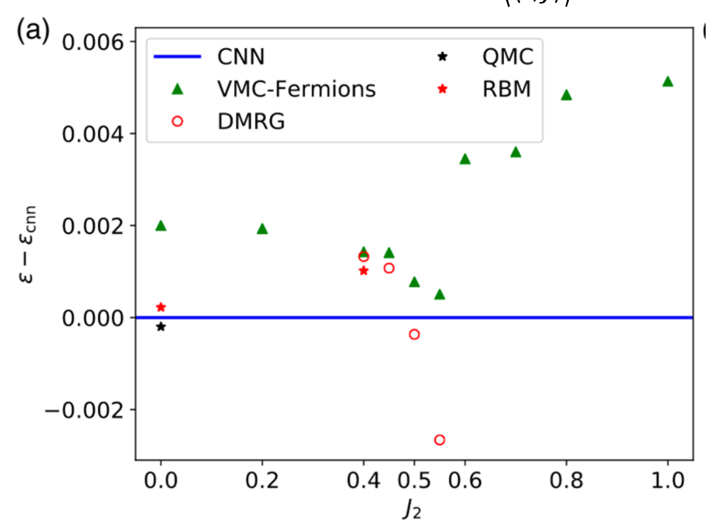
[G Carleo and M Troyer, Science (2017)]

Feed-forward architectures: Convolutional NN (CNN)



Convolutional filters: Capitalize on translational symmetries

$$H = J_1 \sum_{\langle i,j \rangle} S_i \cdot S_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} S_i \cdot S_j$$



[K Choo et al., PRB 100 (2019)]

Recurrent neural networks: Autoregressive sampling

$$|\Psi\rangle = \sum_S \exp[i\phi(S)]\sqrt{P(S)}|S\rangle$$

$$\downarrow$$

$$P(S) = |\Psi(S)|^2$$

$$P(S) = P(s_1)P(s_2|s_1)\cdot \ldots \cdot P(s_N|s_{N-1},\ldots,s_2,s_1)$$

Autoregressive trick: Decompose probability distribution P into conditional probabilities

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Sampling recipe:

1. Sample $s_i \sim p(s_i|s_{<i})$ and concatenate to $s_{<i}$
2. Use samples to define $p(s_{i+1}|s_{<i+1})$

Obtain *uncorrelated* samples

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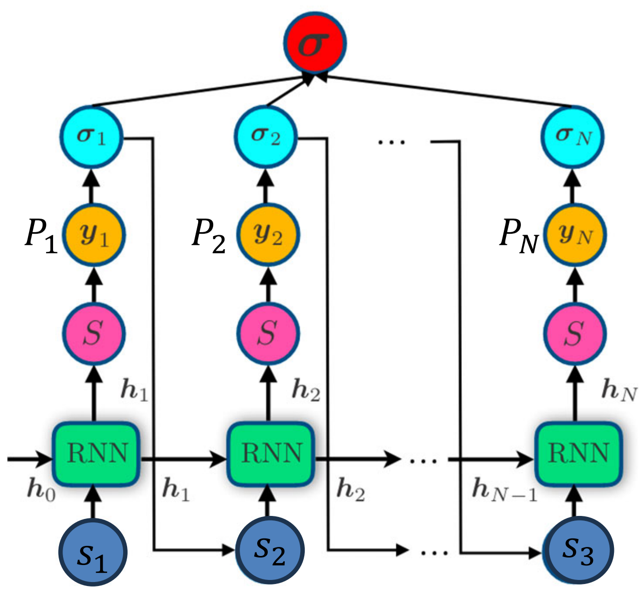

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[M Hibat-Allah et al., PRR 2 (2020)]

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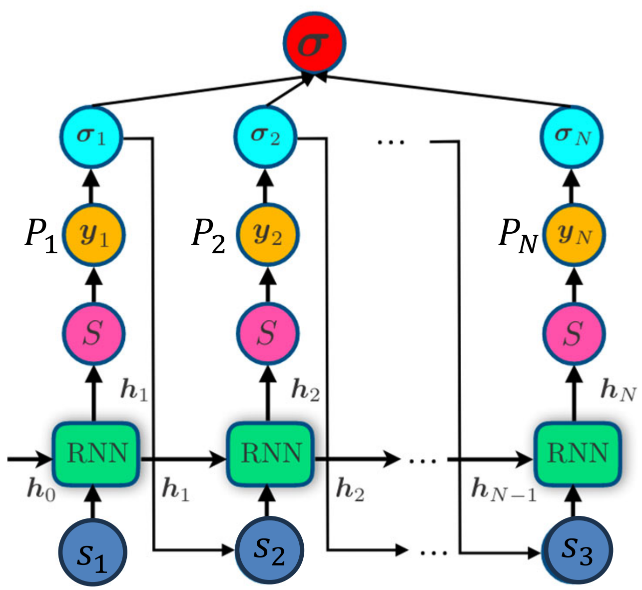
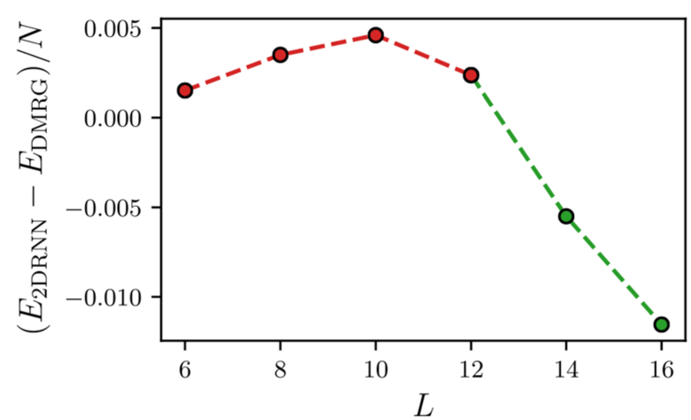


AFH Heisenberg on triangular lattice



[M Hibat-Allah et al., PRR 2 (2020)]
[M Hibat-Allah et al., arXiv:2207.14314 (2022)]

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Recent advances: minSR

Stochastic reconfiguration:

$$\theta_{new} = \theta - \eta S^{-1} \nabla_\theta \mathrm{E}_\theta$$

Quantum geometric tensor

$$S_{kk'} = \left\langle O_k^\dagger O_{k'} \right\rangle - \langle O_k^\dagger \rangle \langle O_{k'} \rangle \quad \text{with} \quad O_k(S) = \frac{\partial \ln \Psi_\theta}{\partial \theta_k}(S)$$

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↓

$[N_{parameters} \times N_{parameters}]$

Limit: GPU memory

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Linear algebra trick:
Instead invert $[N_{samples} \times N_{samples}]$ matrix
"minSR"

[A Chen and M Heyl, arXiv:*2302.01941* (2023)]

[R Rende et al., arXiv:2310.05715 (2023)]

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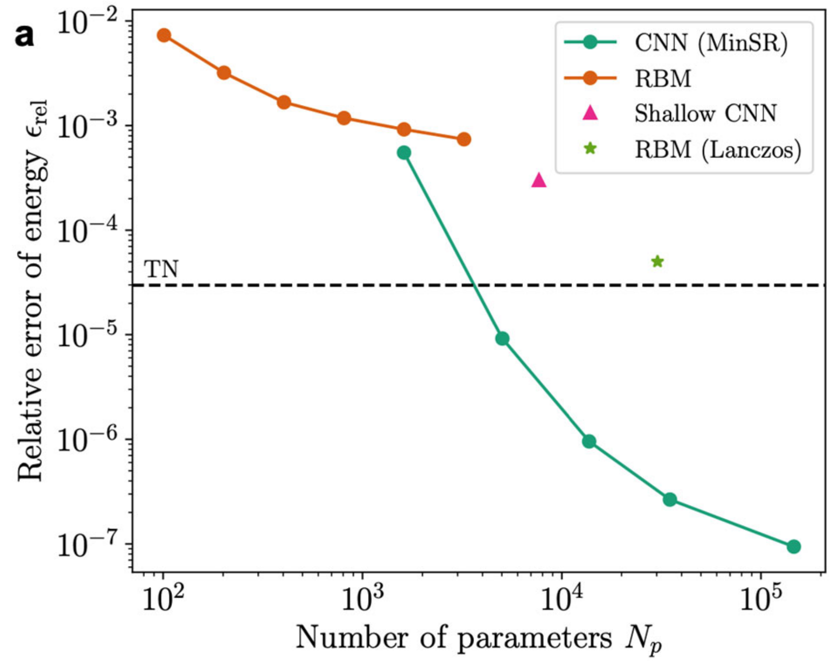

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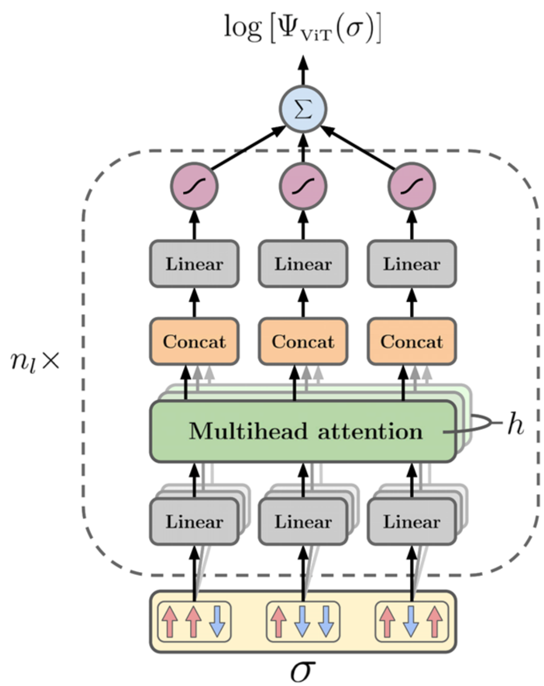
Benchmark energies



Vision transformer

$$\log\Psi(S) = RBM(\mathcal{T}(S))$$

$$H = J_1 \sum_{\langle i,j \rangle} S_i \cdot S_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} S_i \cdot S_j$$

TABLE I. Ground-state energy on the 10×10 square lattice at $J_2/J_1 = 0.5$.

Energy per site	Wave function	# parameters	Marshall prior	Reference	Year
-0.48941(1)	NNQS	893994	Not available	[32]	2023
-0.494757(12)	CNN	Not available	No	[22]	2020
-0.4947359(1)	Shallow CNN	11009	Not available	[21]	2018
-0.49516(1)	Deep CNN	7676	Yes	[20]	2019
-0.495502(1)	PEPS + Deep CNN	3531	No	[33]	2021
-0.495530	DMRG	8192 SU(2) states	No	[31]	2014
-0.495627(6)	aCNN	6538	Yes	[34]	2023
-0.49575(3)	RBM-fermionic	2000	Yes	[15]	2019
-0.49586(4)	CNN	10952	Yes	[35]	2023
-0.4968(4)	RBM ($p = 1$)	Not available	Yes	[36]	2022
-0.49717(1)	Deep CNN	106529	Yes	[28]	2022
-0.497437(7)	GCNN	Not available	No	[27]	2021
-0.497468(1)	Deep CNN	421953	Yes	[30]	2022
-0.4975490(2)	VMC ($p = 2$)	5	Yes	[13]	2013
-0.497627(1)	Deep CNN	146320	Yes	[29]	2023
-0.497629(1)	RBM+PP	5200	Yes	[37]	2021
-0.497634(1)	**Deep ViT**	**267720**	**No**	**Present work**	**2023**

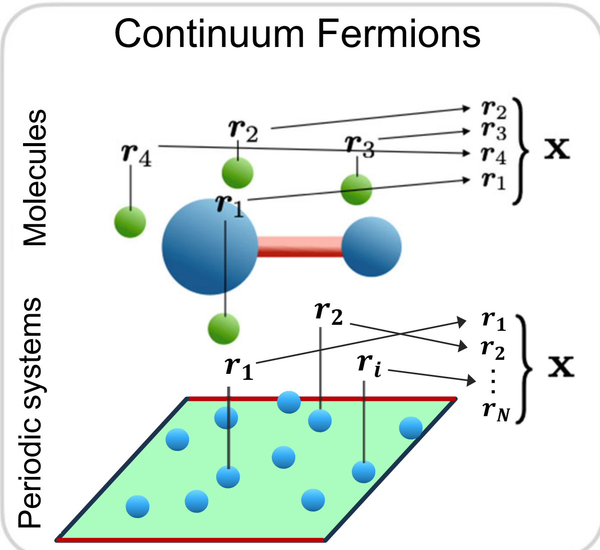
[R Rende et al., arXiv:2310.05715 (2023)]

Part IV
Fermionic systems

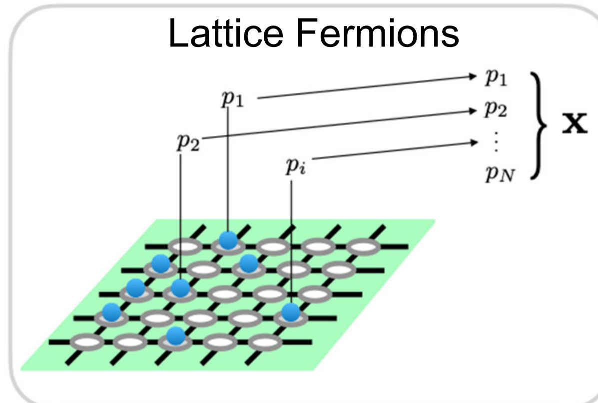
Fermions

[C Kenny et al., *Nature communications* 11.1 (2020)]

[J Robledo Moreno et al., PNAS 119 (32)]



Molecules, solids (full ab-initio), electron gas, …



Effective lattice models of materials (Hubbard-like models)

Quantum simulators: Ultracold atoms

[M Medvidovic, J Robledo Moreno, arXiv:2402.11014 (2024)]

Fermions (continuum)

Fermions (continuum)

First quantization

Second quantization

Antisymmetry

$$\Psi(r_1, r_2, \dots r_N) = -\Psi(r_2, r_1, \dots r_N)$$

$$\{\hat{c}_\alpha^\dagger, \hat{c}_\beta^\dagger\} = \{\hat{c}_\alpha, \hat{c}_\beta\} = 0, \ \{\hat{c}_\alpha, \hat{c}_\beta^\dagger\} = \delta_{\alpha\beta},$$

↓

Creates single-particle state $|\phi_\alpha\rangle$,
e.g. $\phi_\alpha(\boldsymbol{r_i}) = e^{i\boldsymbol{k_\alpha}\cdot\boldsymbol{r}_i}$

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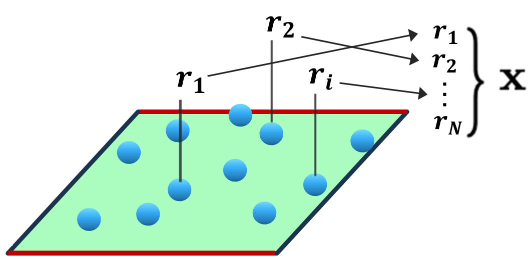
Fermions (continuum)

Antisymmetry

Choice of basis

First quantization

$$\Psi(r_1, r_2, \dots r_N) = -\Psi(r_2, r_1, \dots r_N)$$



$\Psi(\boldsymbol{x})$ needs to be constructed such that antisymmetry is fulfilled

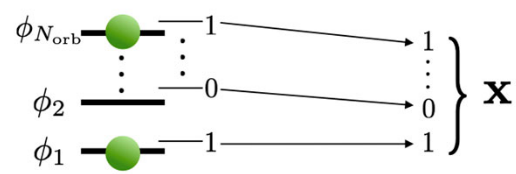
e.g. using a **Slater determinant**

Second quantization

$$\left\{\hat{c}_\alpha^\dagger, \hat{c}_\beta^\dagger\right\} = \left\{\hat{c}_\alpha, \hat{c}_\beta\right\} = 0, \ \left\{\hat{c}_\alpha, \hat{c}_\beta^\dagger\right\} = \delta_{\alpha\beta},$$

↓

Creates single-particle state $|\phi_\alpha\rangle$,
e.g. $\phi_\alpha(\boldsymbol{r_i}) = e^{i\boldsymbol{k_\alpha}\cdot\boldsymbol{r}_i}$



$\Psi(\boldsymbol{x})$ antisymmetric by construction of $\boldsymbol{x}$

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Variational wave-functions for fermions (continuum)

Non-interacting wave-function (Hartree-Fock):

$$\Psi(r_1, \dots r_N) = \frac{1}{\sqrt{N!}} \mathrm{Det}(\{\phi_\alpha(r_i)\}_{\alpha i})$$

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Slater-Jastrow-Backflow wave-functions

Include correlations:

$$\Psi(r_1, \dots r_N) = e^{-J(\boldsymbol{r}_1, \dots \boldsymbol{r}_N)} \frac{1}{\sqrt{N!}} \text{Det}\left(\{\phi_j(\tilde{\boldsymbol{r}}_i)\}_{ij} \right)$$

Jastrow factor

$$J(\boldsymbol{r}_1, \dots \boldsymbol{r}_N) = \sum_{ij} J_{ij}(|\boldsymbol{r}_i - \boldsymbol{r}_j|)$$

Slater-Jastrow-Backflow wave-functions

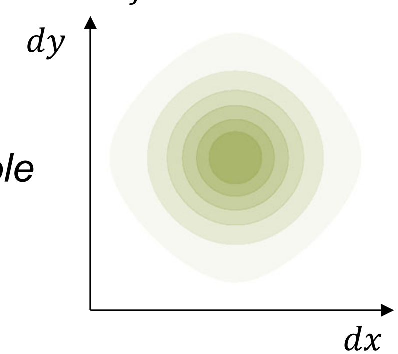
Include correlations:

$$\Psi(r_1, \dots r_N) = e^{-J(r_1, \dots r_N)} \frac{1}{\sqrt{N!}} \text{Det}\left(\{\phi_j(\tilde{r}_i)\}_{ij} \right)$$

Jastrow factor

$$J(\boldsymbol{r}_1, \dots \boldsymbol{r}_N) = \sum_{ij} J_{ij}(|\boldsymbol{r}_i - \boldsymbol{r}_j|)$$

Creates correlation hole



Slater-Jastrow-Backflow wave-functions

Include correlations:

$$\Psi(r_1, \dots r_N) = e^{-J(r_1,\dots r_N)} \frac{1}{\sqrt{N!}} \mathrm{Det}\left(\{\phi_j(\tilde{r}_i)\}_{ij} \right)$$

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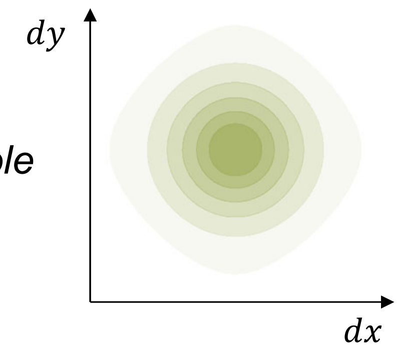


Backflow transformation

$$\tilde{\boldsymbol{r}}_i = \boldsymbol{r}_i + \sum_j \xi_{\tau_i,\tau_j}(|\boldsymbol{r}_i - \boldsymbol{r}_j|)(\boldsymbol{r}_j - \boldsymbol{r}_i)$$

Slater-Jastrow-Backflow wave-functions

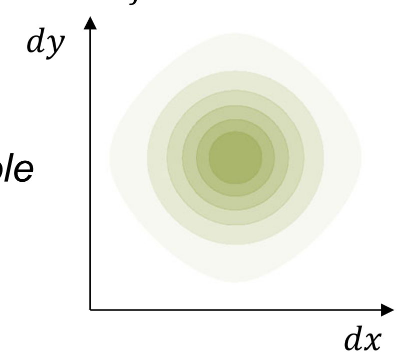
Include correlations:

$$\Psi(r_1, \dots r_N) = \color{red}{e^{-J(r_1, \dots r_N)}} \frac{1}{\sqrt{N!}} \text{Det}\left(\{\phi_j(\color{red}{\tilde{\boldsymbol{r}}_i})\}_{ij} \right)$$

Jastrow factor

$$J(\boldsymbol{r}_1, \dots \boldsymbol{r}_N) = \sum_{ij} J_{ij}(|\boldsymbol{r}_i - \boldsymbol{r}_j|)$$

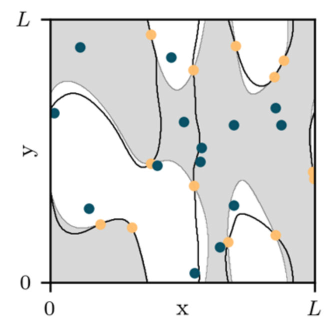
Creates correlation hole



Backflow transformation

$$\tilde{\boldsymbol{r}}_i = \boldsymbol{r}_i + \sum_j \xi_{\tau_i, \tau_j}(|\boldsymbol{r}_i - \boldsymbol{r}_j|)(\boldsymbol{r}_j - \boldsymbol{r}_i)$$

Modifies nodal structure



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Slater-Jastrow-Backflow wave-functions

Include correlations:

$$\Psi(r_1, \dots r_N) = e^{-J(r_1,\dots r_N)} \frac{1}{\sqrt{N!}} \text{Det}\left(\{\phi_j(\tilde{\boldsymbol{r}}_i)\}_{ij} \right)$$

Jastrow factor

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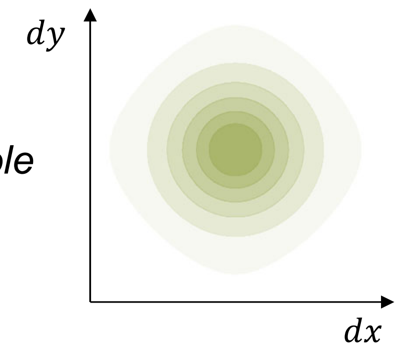
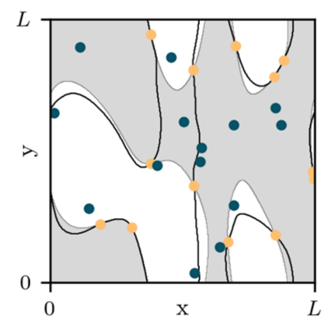


Backflow transformation

$$\tilde{\boldsymbol{r}}_i = \boldsymbol{r}_i + \sum_j \xi_{\tau_i,\tau_j}(|\boldsymbol{r}_i - \boldsymbol{r}_j|)(\boldsymbol{r}_j - \boldsymbol{r}_i)$$

Modifies nodal structure

can in principle represent generic wave-functions



Slater-Jastrow-Backflow wave-functions

Include correlations:

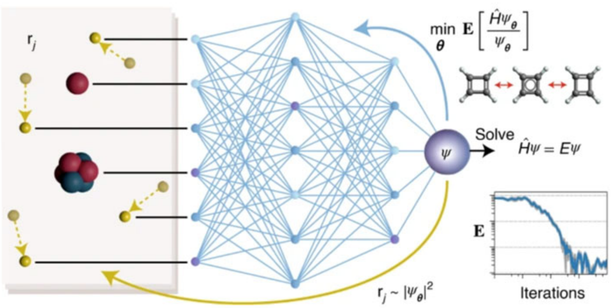
$$\Psi(r_1, \dots r_N) = e^{-J(r_1,\dots r_N)} \frac{1}{\sqrt{N!}} \text{Det}\left(\{\phi_j(\tilde{r}_i)\}_{ij} \right)$$

Backflow transformation

$$\tilde{r}_i = r_i + \underbrace{\delta r_i(r_1, \dots r_N)}$$

Parametrize with NN

[J Hermann et al., Nature Chemistry 12 (2020)]



Slater-Jastrow-Backflow wave-functions

Include correlations:

$$\Psi(r_1, \dots r_N) = e^{-J(r_1, \dots r_N)} \frac{1}{\sqrt{N!}} \mathrm{Det}\left(\{\phi_j(\tilde{r}_i)\}_{ij} \right)$$

Iterative backflow [D Luo and B Clark, PRL 122 (2019)]

FermiNet [D Pfau et al., PRR 2 (2020)]
[JS Spencer et al., arxiv:2011.07125 (2020)]

PauliNet [J Hermann et al., Nature Chemistry 12 (2020)]

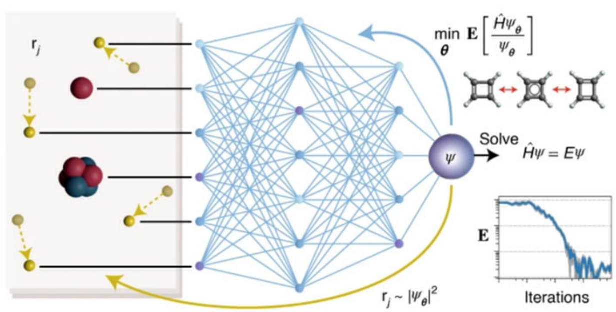
Graph NN [Pescia et al.,arXiv:2305.07240 (2023)]

Backflow transformation

$$\tilde{r}_i = r_i + \delta r_i(r_1, \dots r_N)$$

Parametrize with NN

[J Hermann et al., Nature Chemistry 12 (2020)]

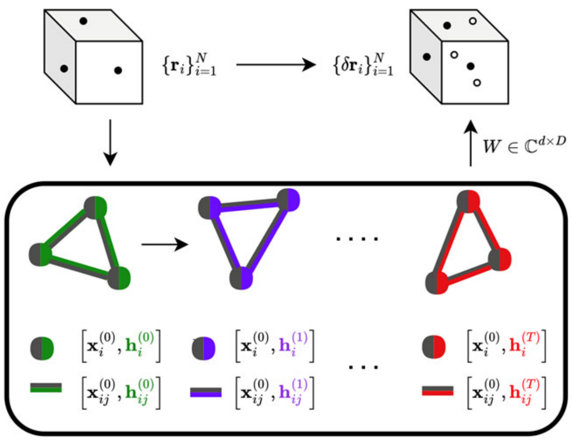


Example: 3D electron gas with message-passing NQS

Backflow transformation

$$\tilde{\boldsymbol{r}}_i = \boldsymbol{r}_i + \delta \boldsymbol{r}_i(\boldsymbol{r}_1, \ldots \boldsymbol{r}_N)$$

Parametrize with Message-passing Graph NN

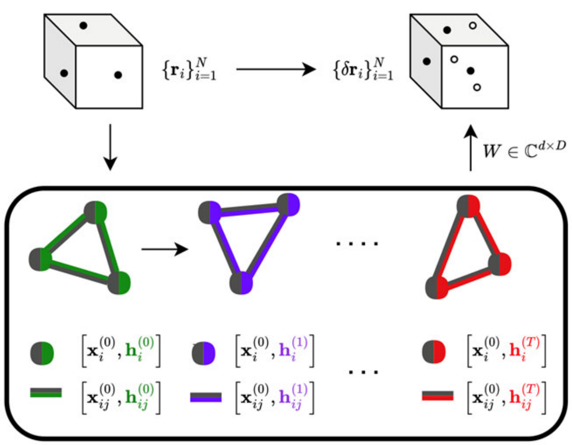


Example: 3D electron gas with message-passing NQS

Backflow transformation

$$\tilde{\boldsymbol{r}}_i = \boldsymbol{r}_i + \delta \boldsymbol{r}_i(\boldsymbol{r}_1, \ldots \boldsymbol{r}_N)$$

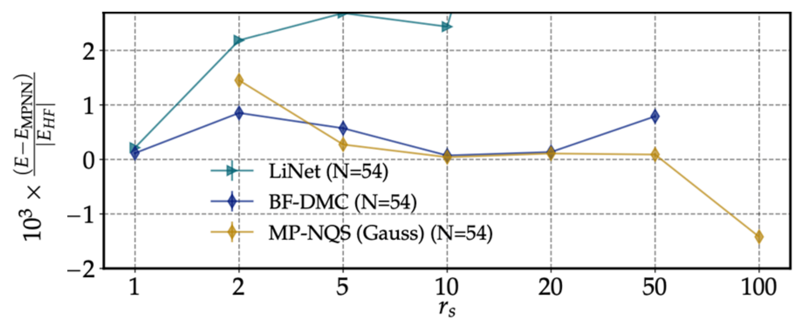
Parametrize with Message-passing Graph NN



3D interacting Electron gas

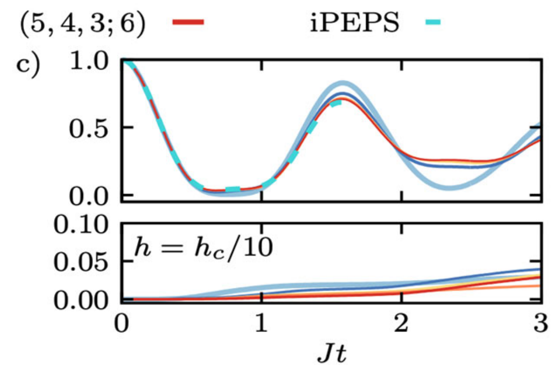
$$H = -\frac{1}{2r_s^2}\sum_i \nabla_i^2 + \frac{1}{r_s}\sum_{i<j}\frac{1}{\left\|r_i - r_j\right\|}$$

Wigner-Seitz radius $r_s = \sqrt[3]{3/(4\pi n)}$



[Pescia et al.,arXiv:2305.07240 (2023)]

Outlook: Other applications



Real time evolution

[M Schmitt and M Heyl, PRL 125 (2020)]
[M Medvidovic and D Sels, PRX Quantum 4 (2023)]
[A Sinibaldi et al., arXiv2305.14294 (2023)]

Open systems

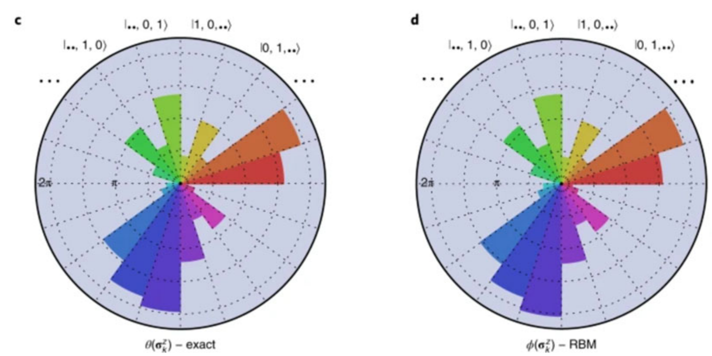
[A. Nagy and V. Savona, Phys. Rev. Lett. 122 (2019)]
[J. Carrasquilla et al., Nature Machine Intelligence 1 (2019)]
[F Vicentini et al., arXiv:2206.13488 (2022)]

Finite temperature

[N Irikura and Hiroki Saito, PRR 2 (2020)]
[Y Nomura et al., PRL 127 (2021)]

Quantum state reconstruction

[G Torlai et al., Nature physics 14 (2018)]
[J Carrasquilla et al., *Nature Machine Intelligence* 1.3 (2019)]
[S Czischek et al., PRB 105 (2022)]



Thank you.

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