Electronic correlation at the 2P-level:

An intro to diagrammatic extensions of DMFT with applications of DΓA

(spin fluctuations, pseudogap, superconductivity)

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QUAST

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1. *Intro:* DMFT as a starting point

2. Basics of its diagrammatic extensions

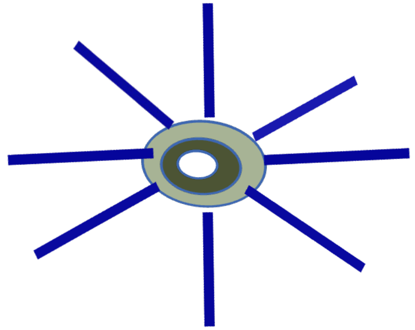
3. How ***nonperturbative*** information is ``encoded'' in the DMFT vertex functions

4. DΓA algorithms: different flavours

5. Pedagogical discussion of relevant results

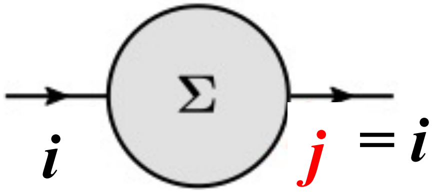
Conclusions & Outlook

DMFT in a nutshell



high connectivity/dimensions

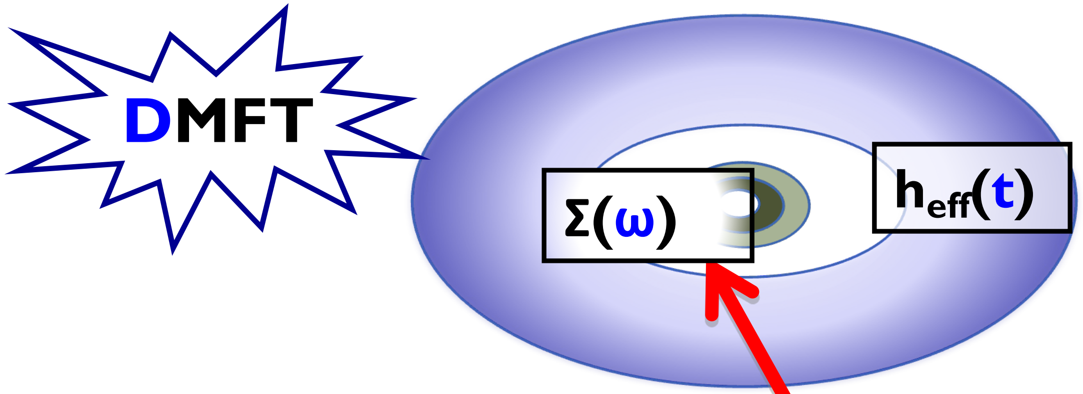
❖ *W. Metzner & D. Vollahardt, PRL (1989)*



$$= \delta_{ij} \Leftrightarrow \Sigma(\vec{k}, \omega) = \Sigma(\omega)$$

the self-energy is **local !!**

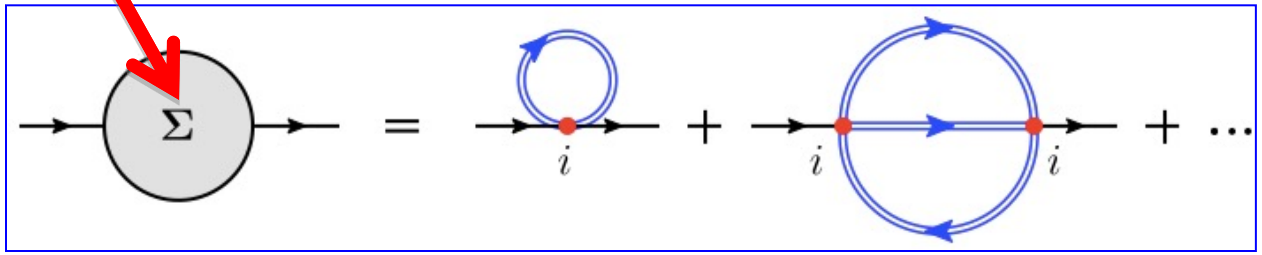
❖ *A.Georges & G. Kotliar, PRB (1992)*



DMFT

=

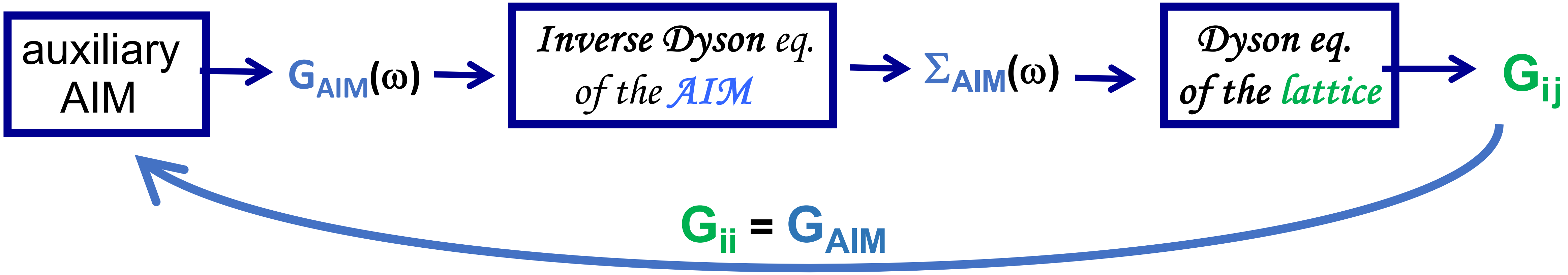
self-consistently adjusted
Anderson Impurity Model (AIM)



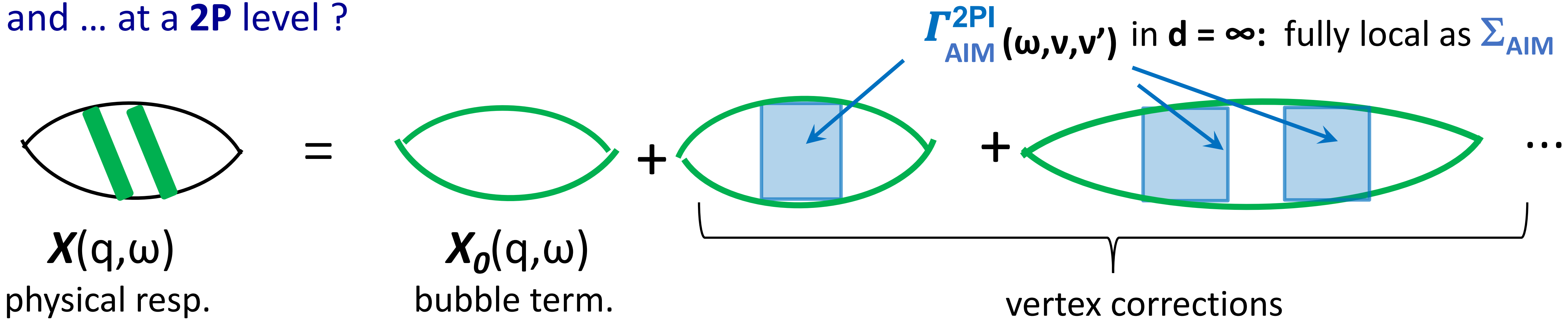
➔ **local,** but **all orders** included!! ➔ **non perturbative** in **U**!!

DMFT algorithm: Self-consistency at the 1P level

★ starting point :



and … at a **2P** level ?



1P- and 2P-self-consistency of DMFT

★ **1P-level :** $G_{ii} = G_{AIM}$ (DMFT self-consistency) ⇒ $\sum_{\vec{k}} G(\vec{k},\nu) = G_{AIM}(\nu)$ ✓ **always** verified in DMFT calculations

d = ∞
DMFT is exact

★ **2P-level :** $\Gamma^{2PI}_{ijkl} = \Gamma^{2PI}_{iiii}\,\delta_{ij}\,\delta_{ik}\delta_{il} = \Gamma^{2PI}_{AIM}$ ⇒ $\sum_{\vec{q}} \chi(\vec{q},\omega) = \chi_{AIM}(\omega)$

$$\text{in } d=\infty : \chi(\vec{q},\omega) \equiv \chi_{AIM}(\omega)\ \forall \vec{q}\ \text{(except special momenta)}$$

❖ *A. Georges et al. RMP (1996)*

1P- and 2P-self-consistency of DMFT

★ **1P-level :** $G_{ij} = G_{AIM}$ (DMFT self-consistency) ⇒ $\sum_{\vec{k}} G(\vec{k},\nu) = G_{AIM}(\nu)$ ✓ **always** verified in DMFT calculations

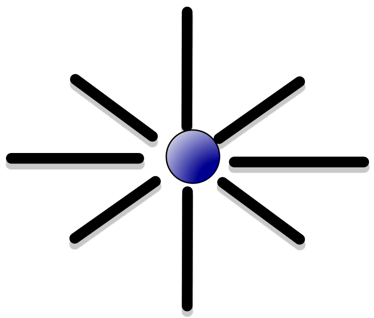
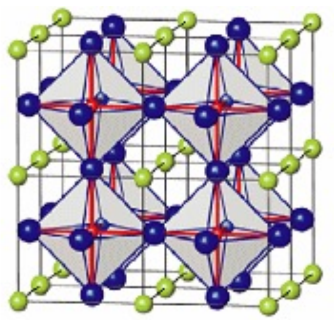
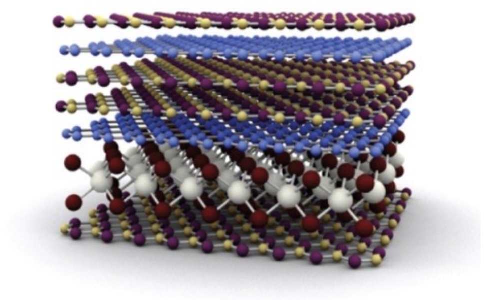
d < ∞
DMFT is an approx.

★ **2P-level :** $\Gamma^{2PI}_{ijkl} \neq \Gamma^{2PI}_{iiii}\,\delta_{ij}\,\delta_{ik}\delta_{il} = \Gamma^{2PI}_{AIM}$ ⇒ $\sum_{\vec{q}} \chi(\vec{q},\omega) \neq \chi_{AIM}(\omega)$

$\chi(\vec{q},\omega) \neq \chi_{AIM}(\omega)\ \forall \vec{q}$ (except special momenta)

- *A. Georges et al. RMP (1996); A. Katanin et al. PRB (2009); G. Rohringer & AT, PRB (2016); L. Del Re & AT, PRB (2021)*

From ∞ dimensions to … „reality“ !

Dimensionality:	Systems:		Spatial correlations **beyond** DMFT:
d = ∞		mathematical **idealization** (DMFT exact!)	None
d = 3		**bulk materials**	in some conditions
d = 2		layered compounds: 2D-networks, adatoms **heterostructures**	quite strong!!

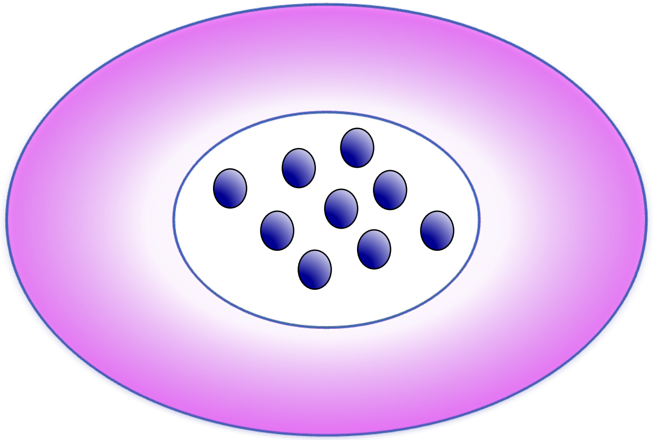
From ∞ dimensions to … „reality“ !

Dimensionality:

Systems:

Spatial correlations **beyond** DMFT:

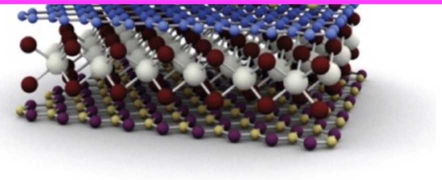
★ **cluster extensions** [⌘ *Kotliar et al. PRL 2001; Huscroft, Jarrell et al. PRL 2001*]



1. **Cellular-DMFT**
   (**C-DMFT:** cluster in real space)

2. **Dynamical Cluster Approx.**
   (**DCA:** cluster in **k**-space)

[⌘ **Review**: “Quantum Cluster Theories”: *Th. Maier, et al.*, RMP *2006*]



adatoms

heterostructures

Outline

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Conclusions & Outlook

Diagrammatic extensions of DMFT

- ★ **Dynamical Vertex Approximation (DΓA)**
  [⌘ ***AT***, *Katanin, Held, PRB (2007)*]
- ★ **Dual Fermion (DF) & Dual Bosons (DB)**
  [⌘ *Rubtsov, Lichtenstein …, PRB (2008); Ann. Phys (2012)*]
- ★ **1Particle Irreducible approach**
  [⌘ *Rohringer,* ***AT*** *et al., PRB (2013)*]
- ★ **DMF^{2}RG**
  [⌘ *Taranto, …,&* ***AT****; PRL (2014); Vilardi ,Taranto & Metzner, PRB (2019)*]
- ★ **TRILEX, QUADRILEX**
  [⌘ *Ayral & Parcollet, PRB 2015; PRB (2016)*]

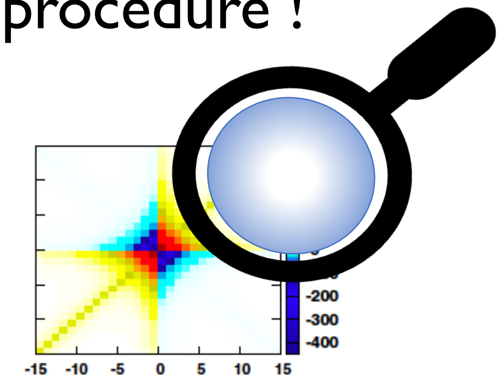
REVIEWS OF MODERN PHYSICS, VOLUME 90, APRIL–JUNE 2018

Diagrammatic routes to nonlocal correlations beyond dynamical mean field theory

Diagrammatic extensions of DMFT

Common feature: **two**-step procedure !

step 1: extract a **local vertex** from DMFT/EDMFT(AIM)



step 2: build upon that the **diagrammatic** expansion

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Diagrammatic routes to nonlocal correlations beyond dynamical mean field theory

Outline

Intro: challenging aspects of many-electron theory

1. DMFT as a starting point

2. Basics of its diagrammatic extensions

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2P-Feynman diagrams: (local) Green's & vertex functions

2P-Green's function: $G^{(2)}_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}(\tau_1,\tau_2,\tau_3,0) = \langle \hat{c}_{\sigma_1}(\tau_1)\hat{c}^\dagger_{\sigma_2}(\tau_2)\hat{c}_{\sigma_3}(\tau_3)\hat{c}^\dagger_{\sigma_4}(0)\rangle$

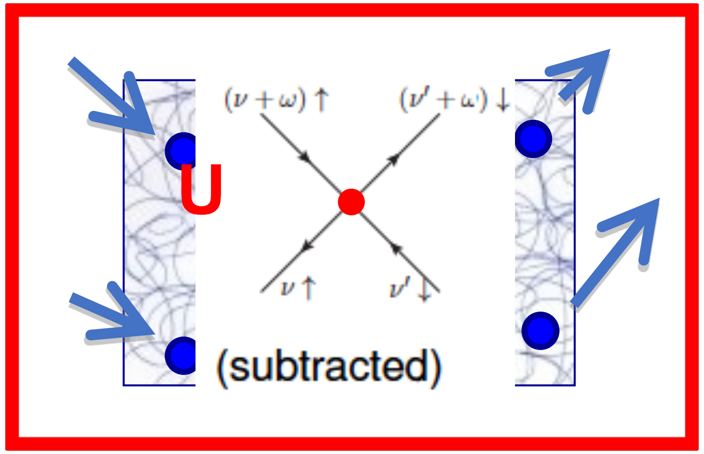
Its Fourier Transform: $G^{(2)}_{\sigma_1\sigma_2\sigma_3\sigma_4}(\omega,\nu,\nu') = \int_0^\beta d\tau_1\, d\tau_2\, d\tau_3\; e^{i\nu\tau_1} e^{-i(\nu+\omega)\tau_2} e^{i(\nu'+\omega)\tau_3}\, G^{(2)}_{\sigma_1\sigma_2\sigma_3\sigma_4}(\tau_1,\tau_2,\tau_3,0)$

➔ computable for AIM

single band: **ED** & **NRG** *[Kugler et al. PRX, (2021)]*

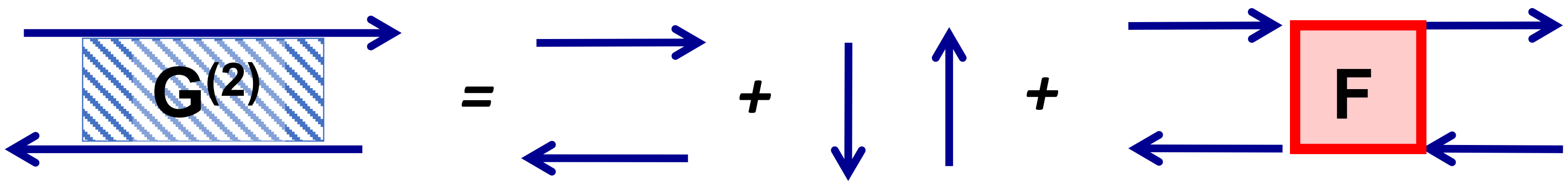
general multi-band case: **CTQMC** *[TRIQS, w2dynamics, ALPS, ….]*

the lowest order :



vertex functions:
2P-analogous of the self-energy

Full vertex
(scattering amplitude)

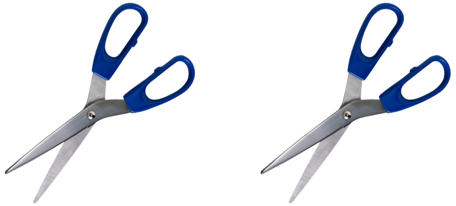


Independent 2P-propagation

= Γ
(fRG notation)

= γ_4
(DF notation)

2P-irreducibility (2PI)

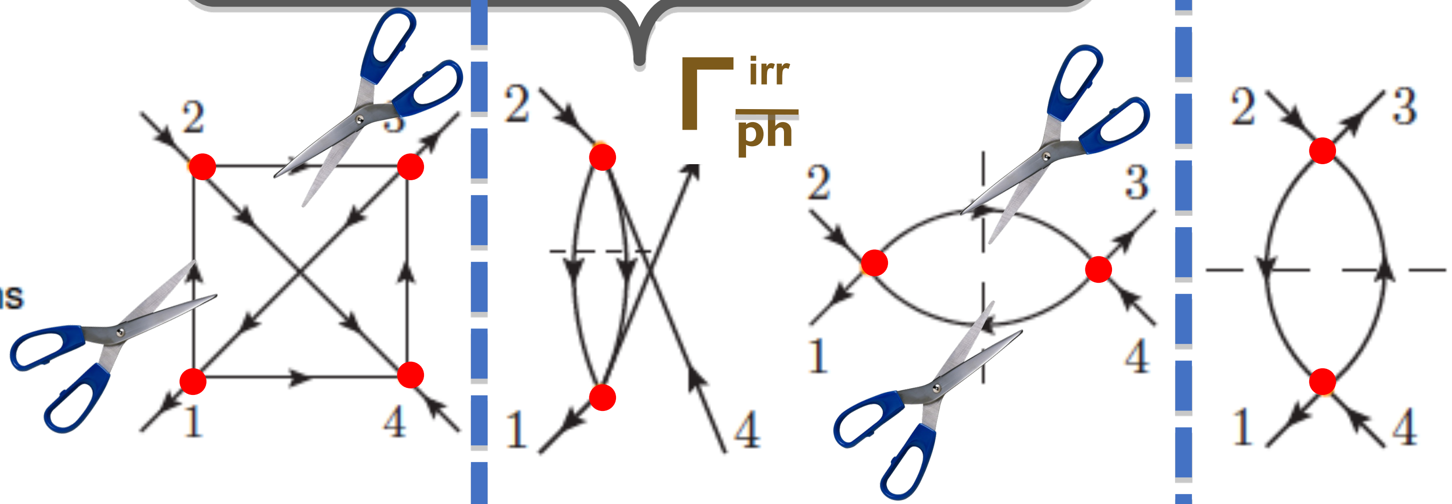


1) parquet equation:

Mr. Parquet ?

truly from parquet floor!

$$F = \underbrace{\Lambda + \Phi_{pp} + \Phi_{ph}}_{\Gamma^{irr}_{\overline{ph}}} + \Phi_{\overline{ph}}$$



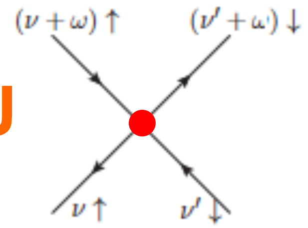
all diagrams	fully irreducible	reducible in particle-particle channel	reducible in particle-hole channel 14-23	reducible in particle-hole channel 13-24

2) Bethe-Salpeter equation (BSE):

e.g., in the *ph transverse* ($\overline{ph}$) channel: $F = \Gamma^{irr}_{\overline{ph}} + \Phi_{\overline{ph}}$

Types of approximations:

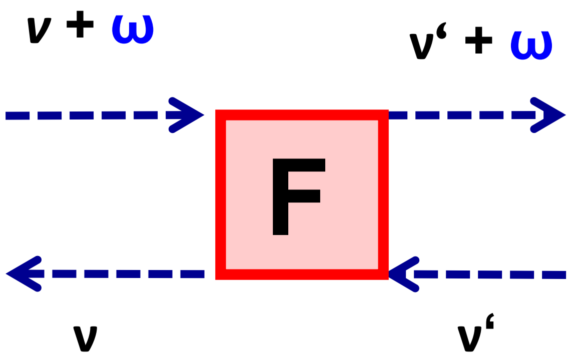
***) LOWEST ORDER (STATIC) APPROXIMATION: U**



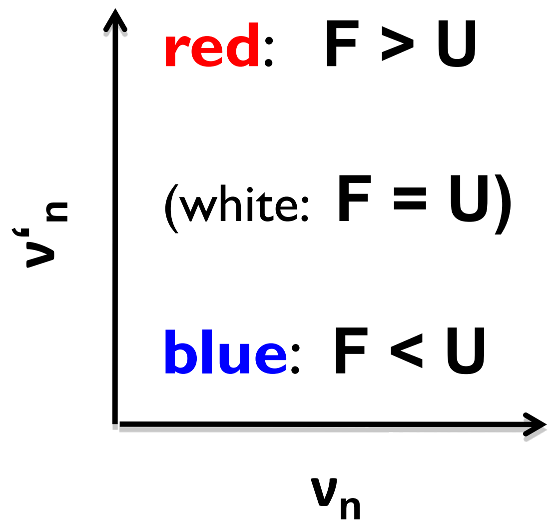
2P- irreducibility

F	e.g.: $F = U$	2^{nd} order perturbation theory
Γ_c	e.g.: $\Gamma_c = U$	RPA, FLEX, pseudopotential parquet
Λ	e.g.: $\Lambda = U$	parquet approximation

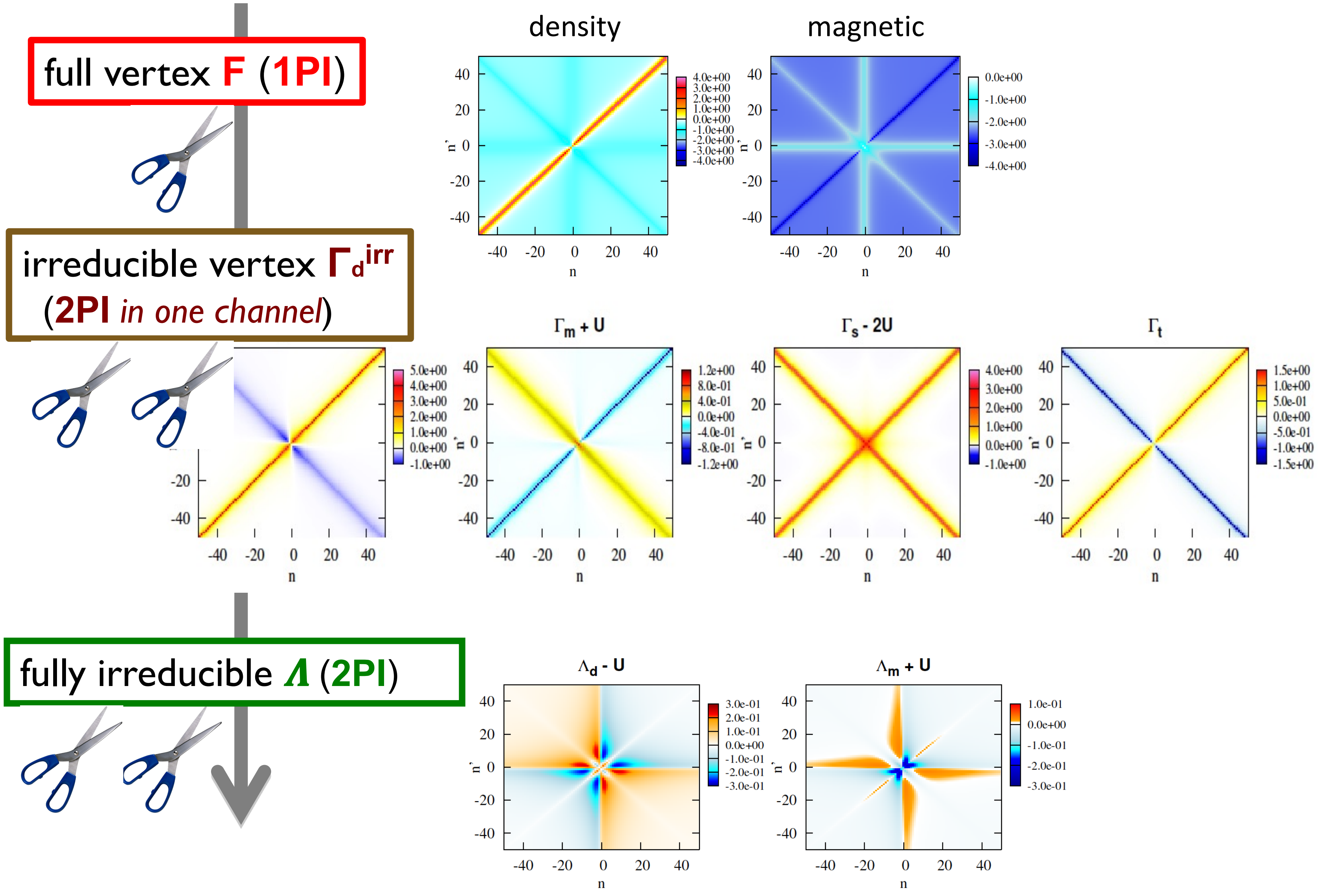
Frequency dependence: an overview



$= F(\nu,\nu',\omega=0)$

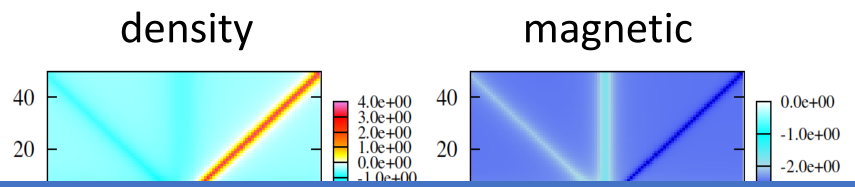


Frequency dependence: an overview



Frequency dependence: an overview

full vertex **F** (1PI)



irreducible ve…
(**2PI** *in one c…*

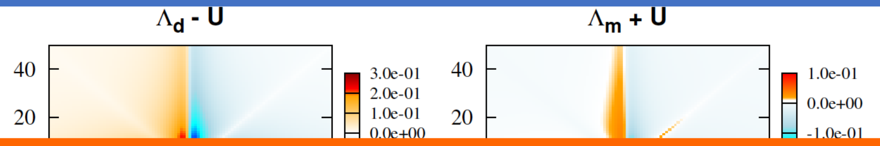
Take-home-message:

 # 1) the more **2PI**, the **easier** the high-frequency dependence

But … *"there is no free lunch"* :

 # 2) the more **2PI**, the **strongest** the low-frequency dependence

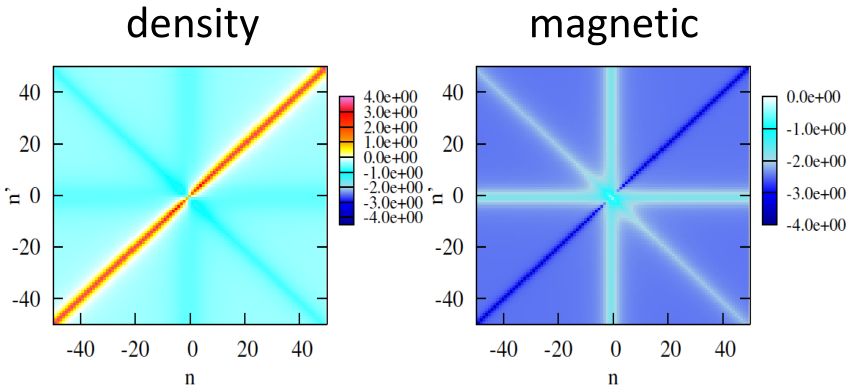
fully irreducible Λ (**2PI**)



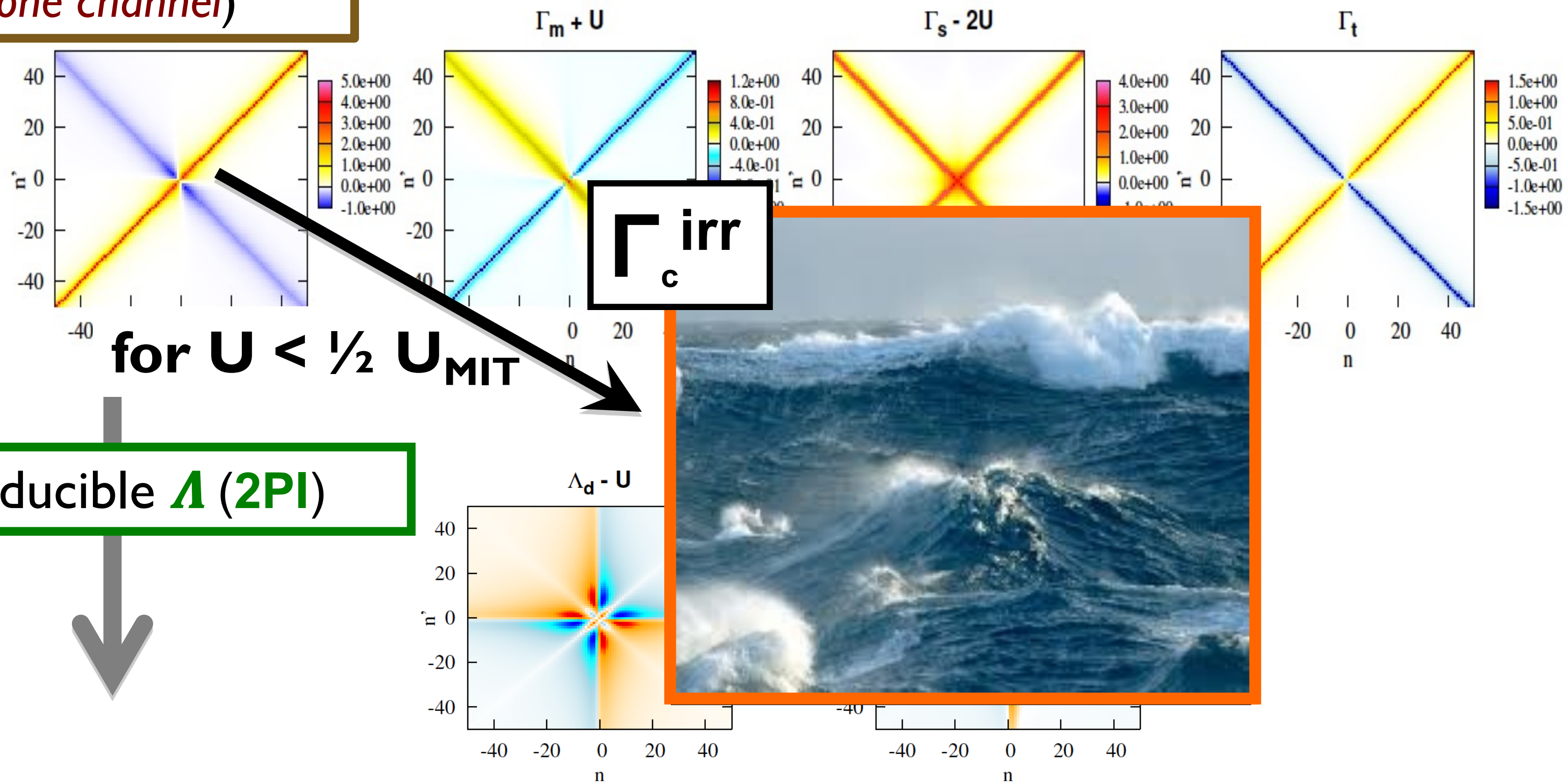
G. Rohringer, A. Valli, & AT, PRB (2012); T. Schäfer, … & AT, PRL (2013)
See also: J. Kunes, PRB (2011); A. Dolfen, PhD-Thesis (2009); D. Luitz, PhD-Thesis (2013); H. Hafermann PRB (2014); M. Kinza PRB (2014); **N. Wentzell, G. Li, PRB (2020)**

Frequency dependence: an overview

full vertex **Γ** (**1PI**)



irreducible vertex **Γ_c^{irr}**
(**2PI** *in one channel*)

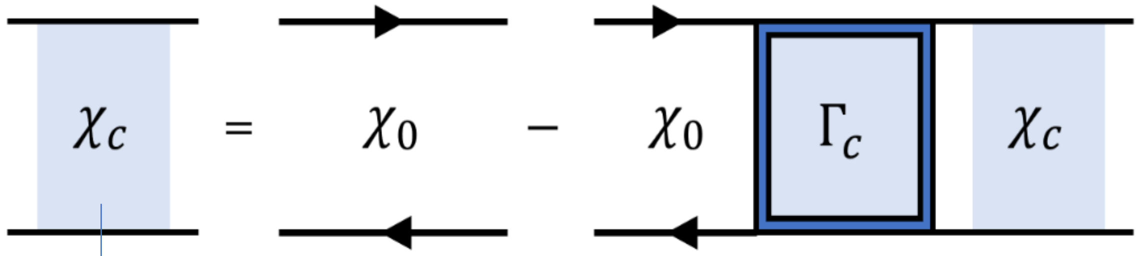


for U < ½ U_{MIT}

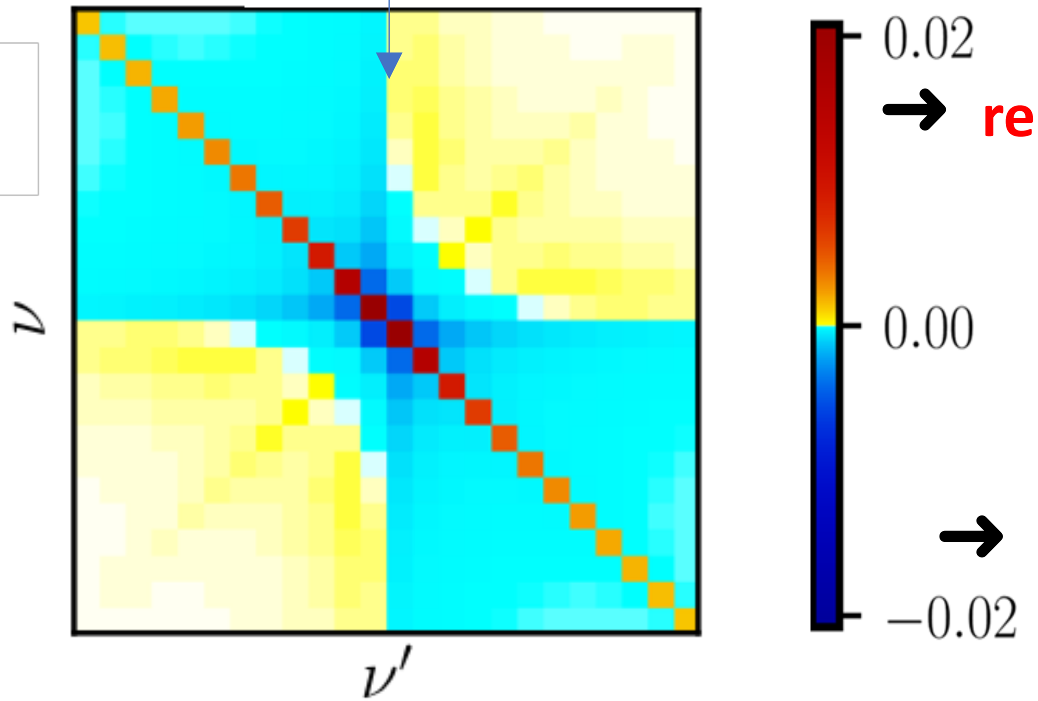
fully irreducible ***Λ*** (**2PI**)

Relation to the physics?

- generalized local charge susceptibility for iΩ=0



$$\chi_c^{phys} = \frac{1}{\beta^2}\sum_{\nu\nu'}$$



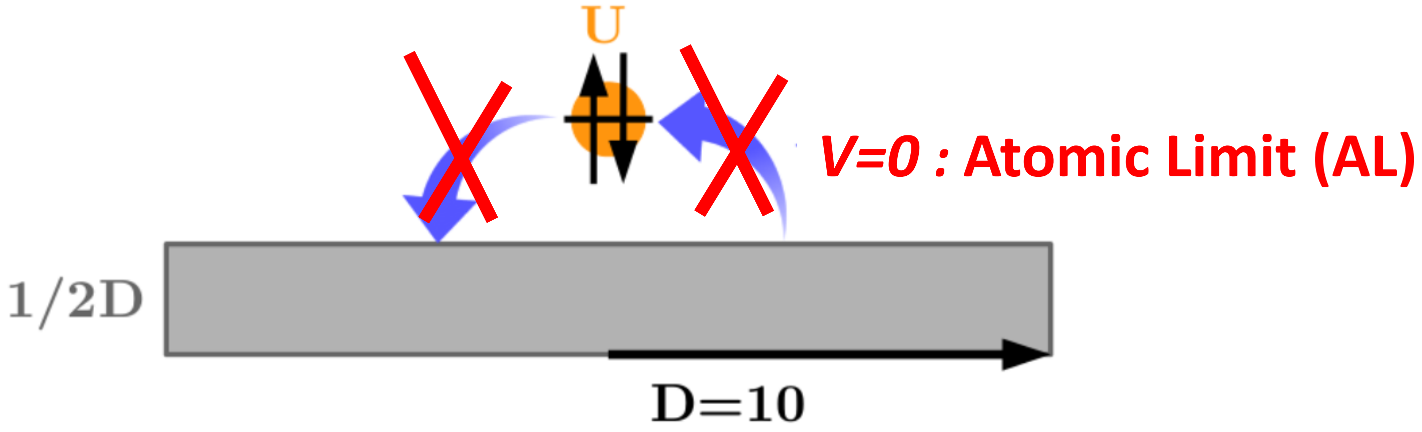
➜ **red = positive values**

➜ **blue = negative values**

❖ P. Chalupa et al., PRL (2021); S. Adler., …, & AT, SciPost Phys. (2024)

Anderson Impurity Model

- wide-band limit, half-filling

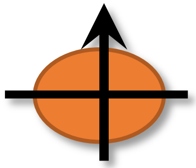


Main physical ingredients:

Curie behavior:

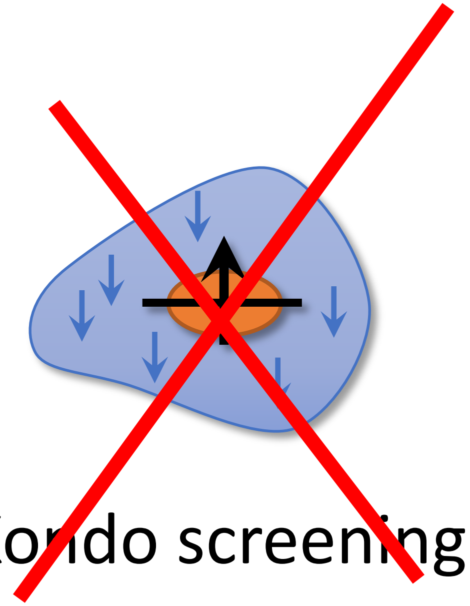
$\chi_m \propto 1/T \Leftrightarrow T\chi_m = \text{const.}$

T < U:



local magnetic moment

T ~ T_K << U:



Kondo screening

Physical response of the AIM

- w2dynamics – CT-HYB

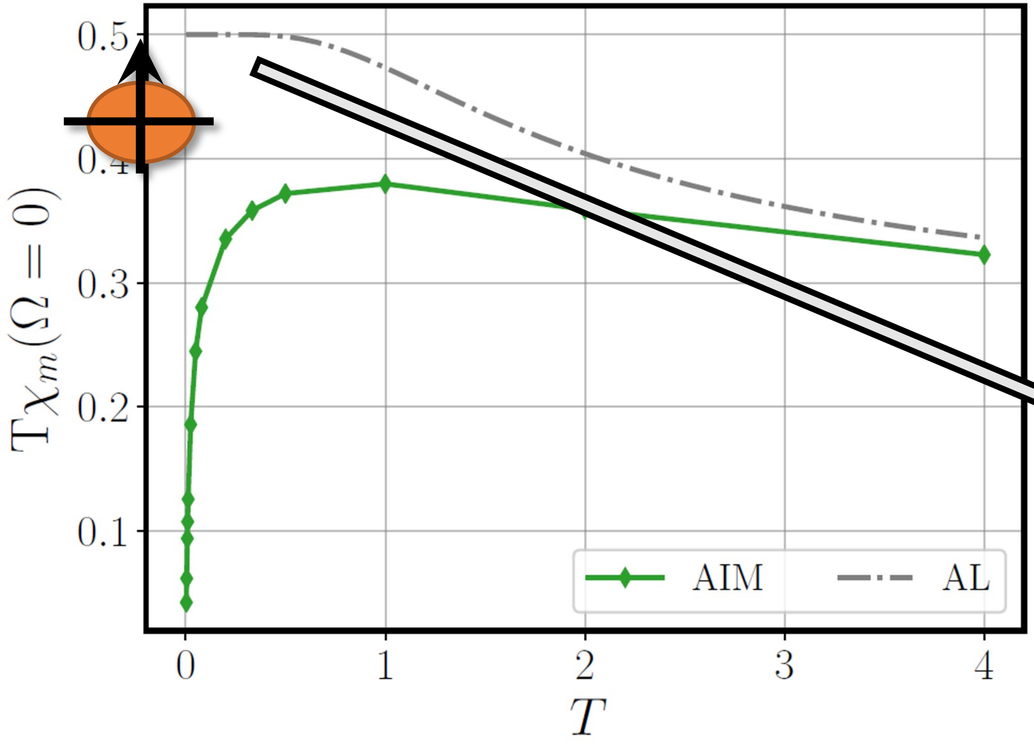
M.Wallerberger, *et.al*, CPC **235,** 388 (2019)

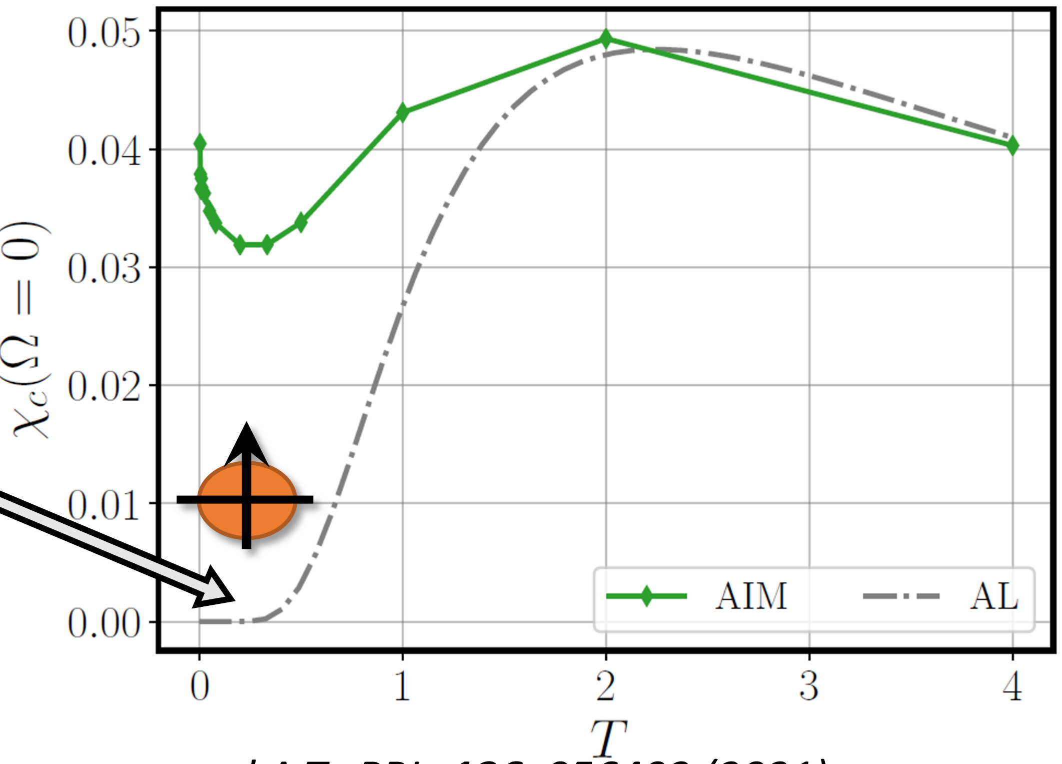
magnetic response

$$\chi_m^{phys} = \frac{1}{\beta^2}\sum_{\nu\nu'}\chi_m^{\nu\nu'}$$

charge response

$$\chi_c^{phys} = \frac{1}{\beta^2}\sum_{\nu\nu'}\chi_c^{\nu\nu'}$$





❖ *P. Chalupa, T. Schäfer, M. Reitner, D. Springer, S. Andergassen, and A.T., PRL* ***126*** *056403 (2021)*

Physical response of AL & AIM

- w2dynamics – CT-HYB

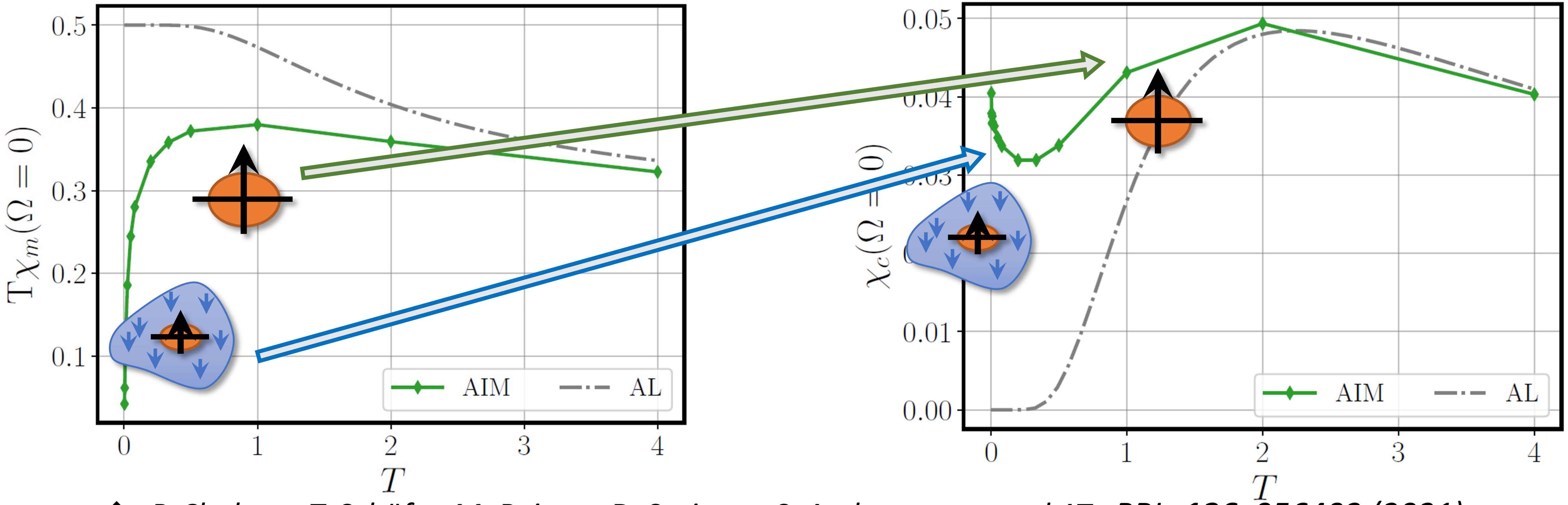
M.Wallerberger, *et.al*, CPC **235,** 388 (2019)

magnetic response

$$\chi_m^{phys} = \frac{1}{\beta^2}\sum_{\nu\nu'}\chi_m^{\nu\nu'}$$

charge response

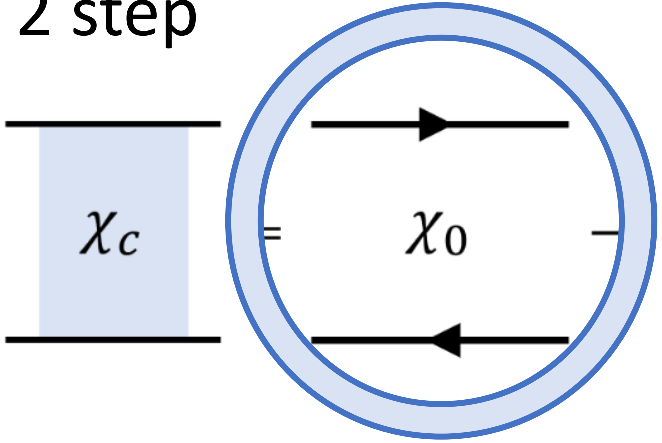
$$\chi_c^{phys} = \frac{1}{\beta^2}\sum_{\nu\nu'}\chi_c^{\nu\nu'}$$

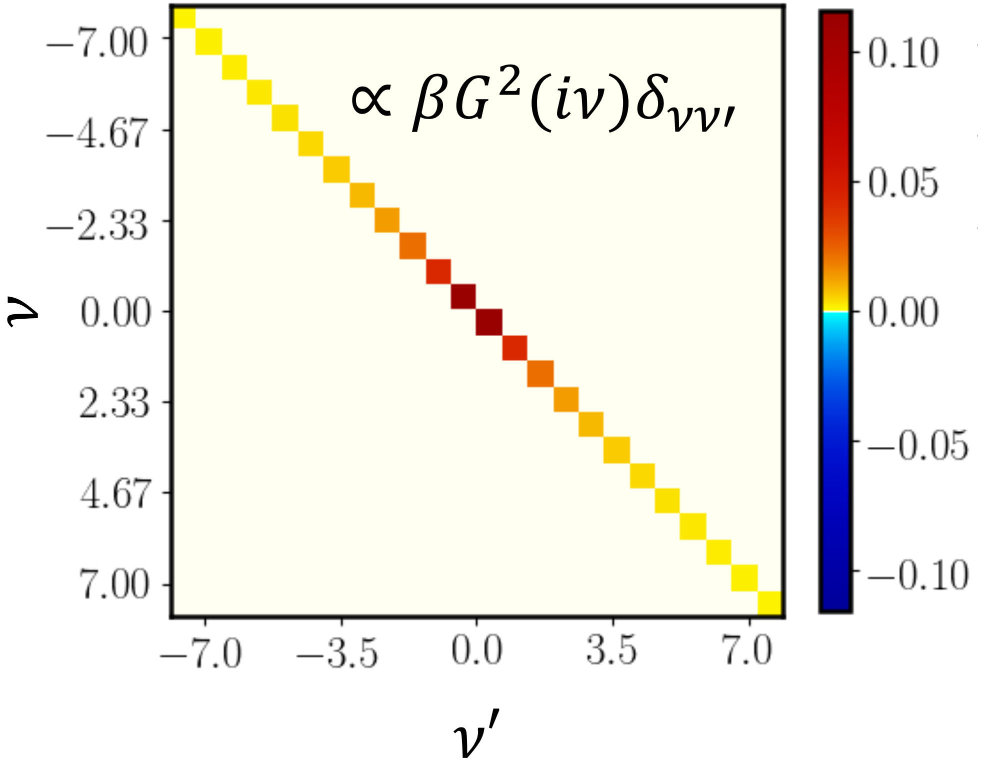


❖ *P. Chalupa, T. Schäfer, M. Reitner, D. Springer, S. Andergassen, and AT., PRL* ***126*** *056403 (2021)*

1.Step: Non interacting case/bubble term

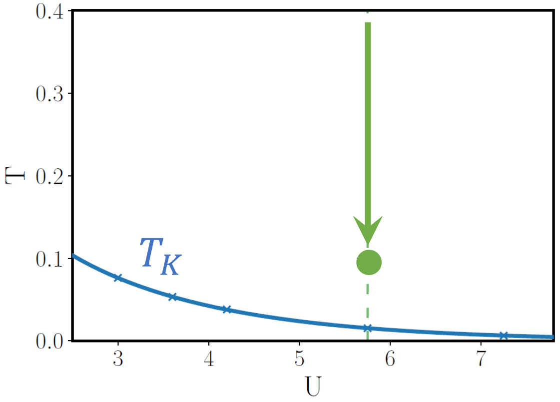
- 2 step





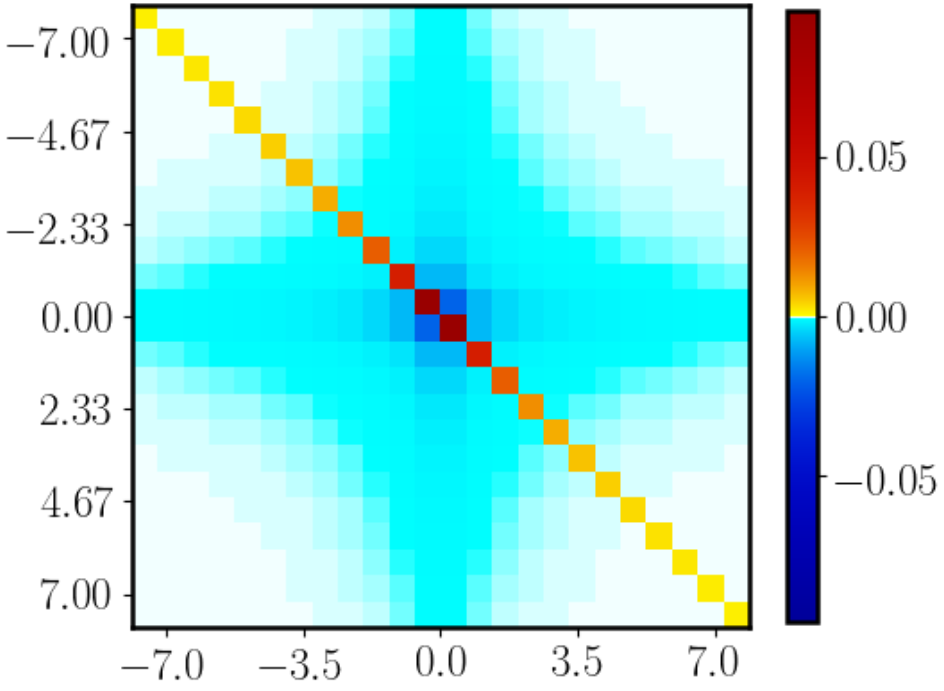
2.Step: weak vs. strong-coupling

e.g.: intermediate temperature region ($\mathbf{T_K < T << U}$)

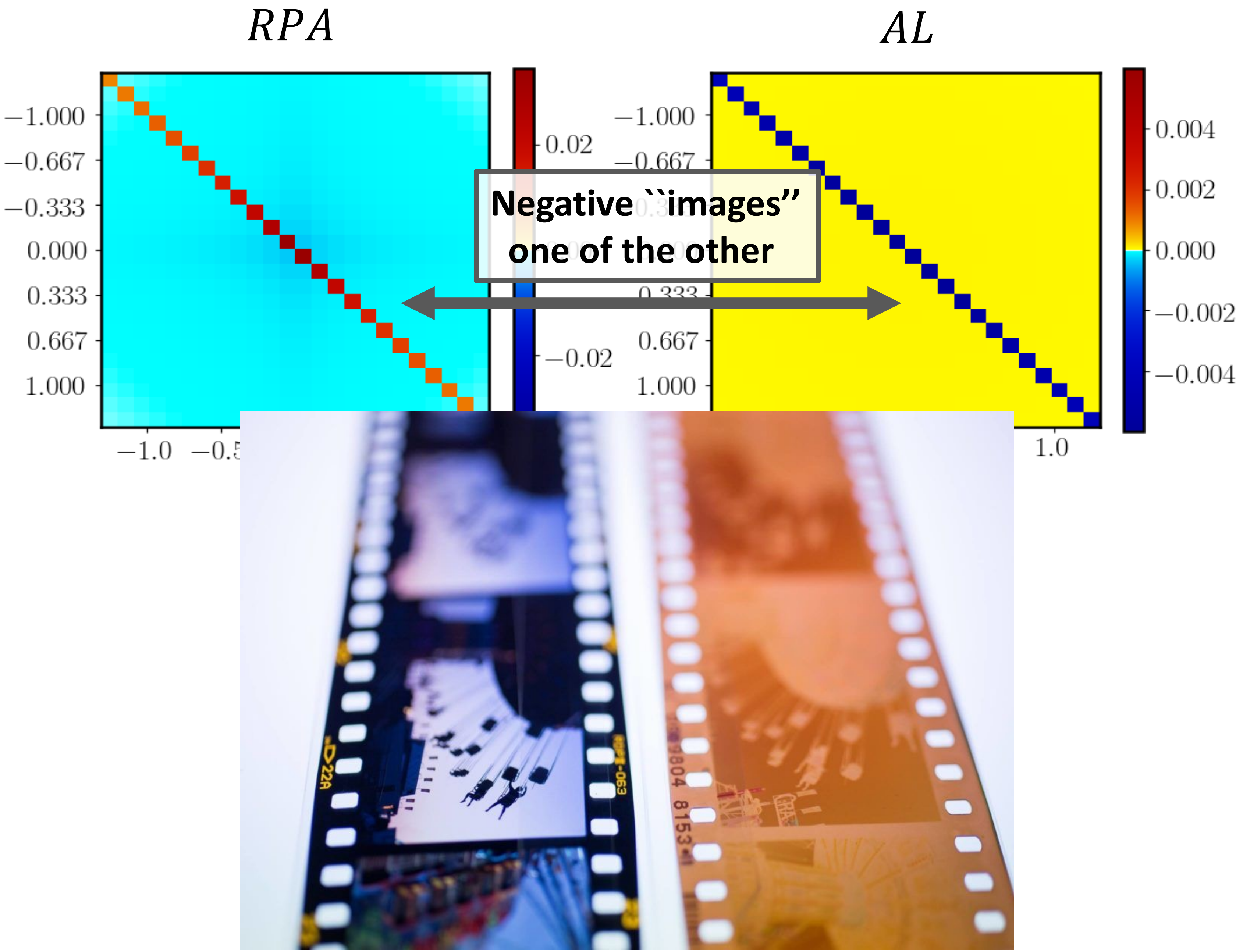


$T_{int} = 0.1$

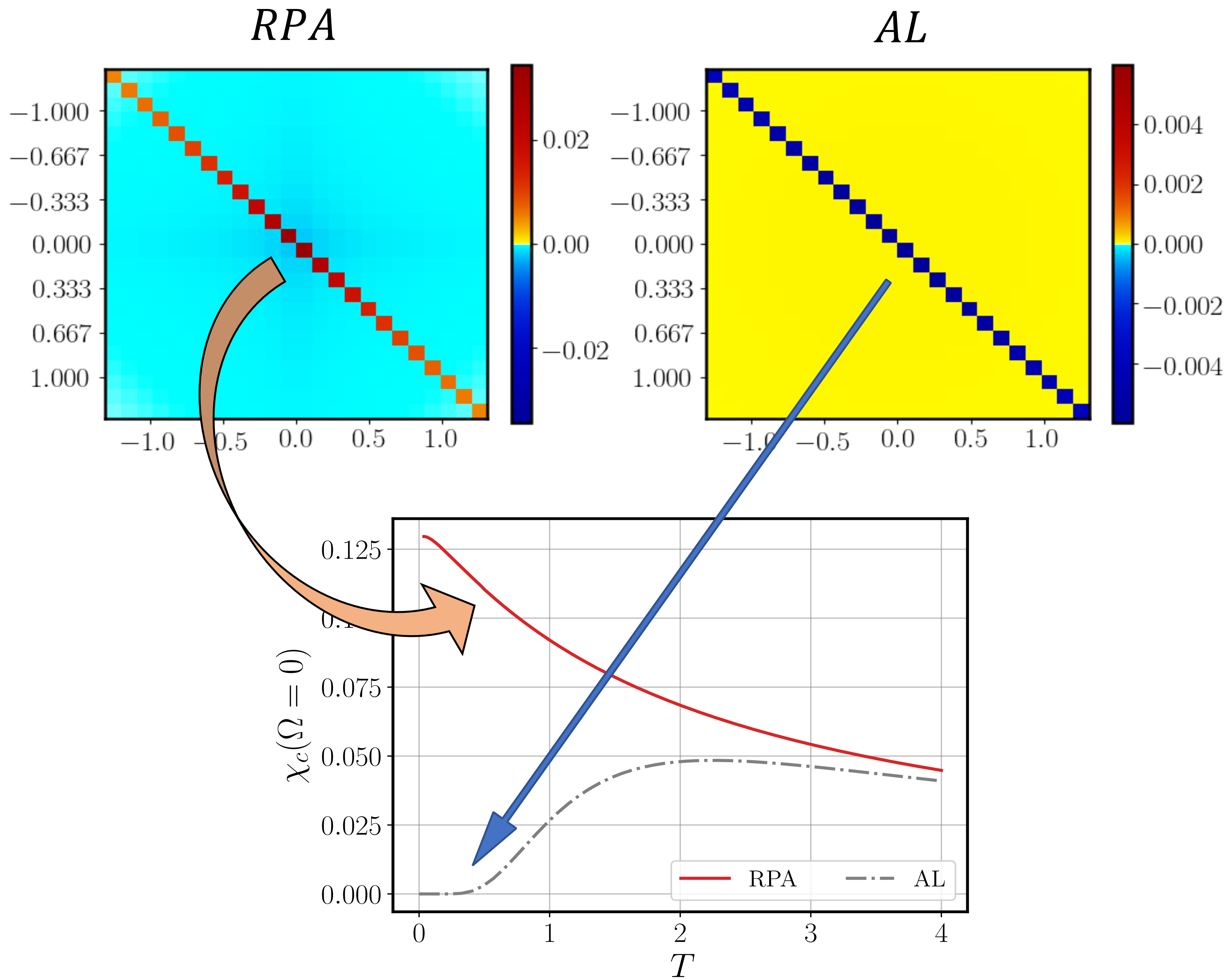
RPA
$(pert.)$



❖ *P. Chalupa, T. Schäfer, M. Reitner, D. Springer, S. Andergassen, and A.T., PRL* ***126*** *056403 (2021)*



RPA
AL
−1.000
−0.667
−0.333
0.000
0.333
0.667
1.000
−1.0
0.02
−0.02
1.0
0.004
0.002
0.000
−0.002
−0.004
Negative ``images''
one of the other
22A
8153

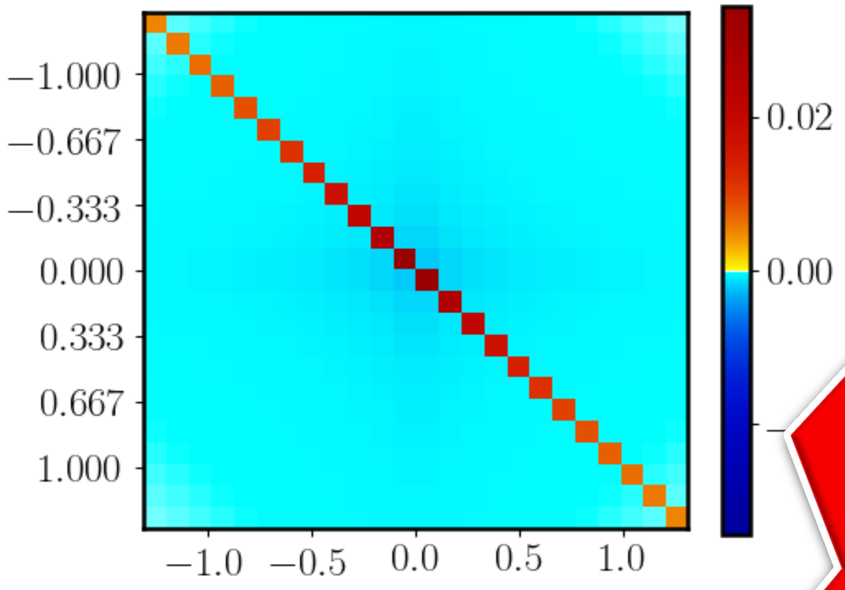


RPA
AL
RPA
AL
$\chi_c(\Omega = 0)$
T

from **weak-** to **strong**-coupling

RPA

(weak-coupling)

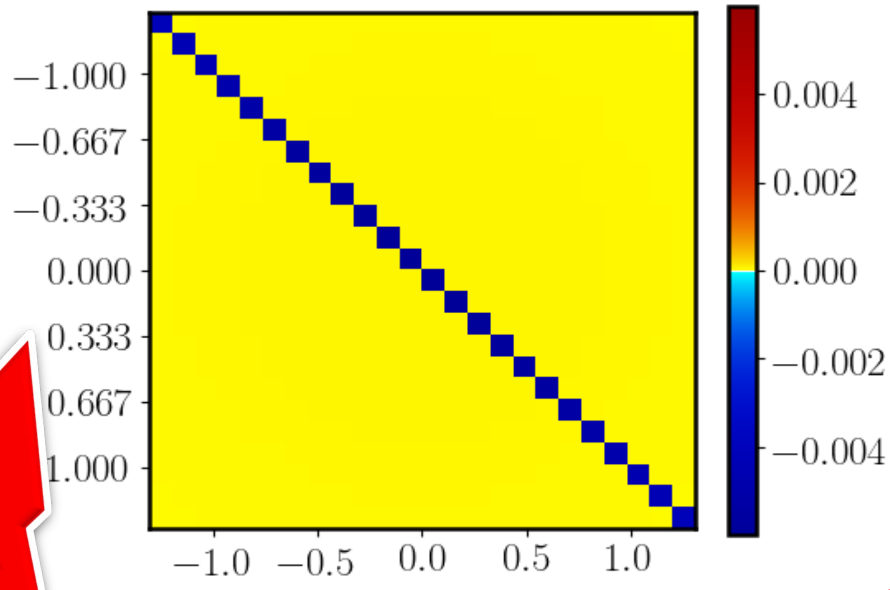


positive diagonal dominates

→ all eigevalues $\lambda > 0$

AL

(strong-coupling)

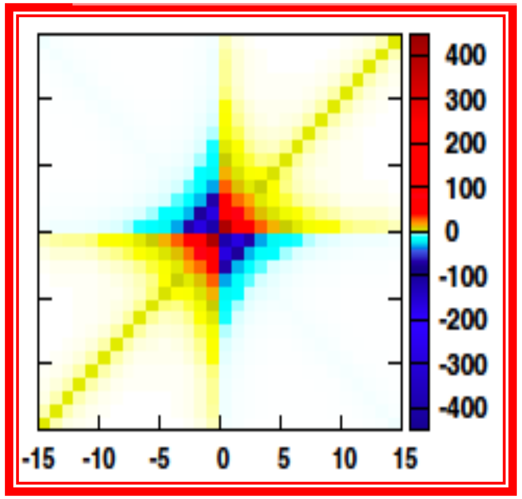


negative diagonal dominates

→ low-freq. eigevalues $\lambda < 0$

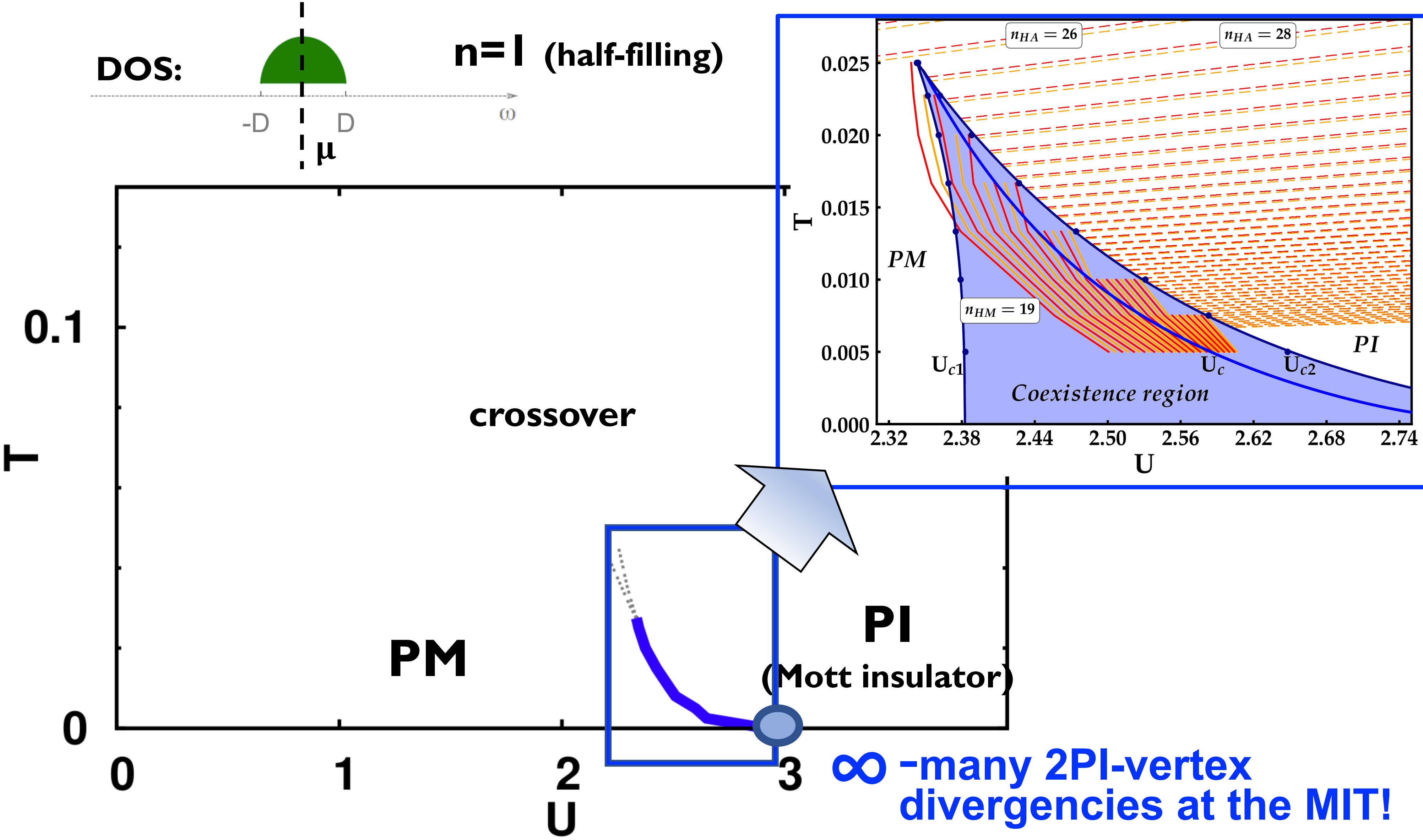
$\lambda = 0$

$\Gamma^{2PI} = \infty$



$$\Gamma_c^{2PI} = \left[\chi_c^{-1} - \chi_0^{-1} \right]$$

Phase diagram of the Hubbard model



∞ -many 2PI-vertex divergencies at the MIT!

MIT data: *N. Bluemer, PhD Thesis*

M. Pelz, ..., & AT, PRB (2023)

Algorithmic challenges

$\Gamma^{2PI} = \infty$

$$\Gamma_c^{2PI} = \frac{\delta^2 \Phi_{LW}}{\delta^2 G}$$

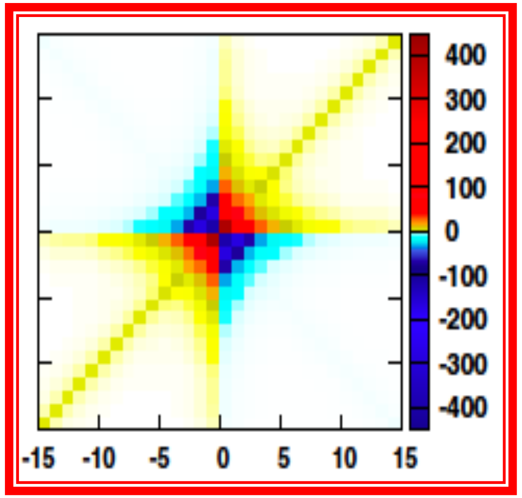
I. Approaches based on 2PI vertices

parquet-based methods

dynamical vertex approximation (**DΓA**)

QUADRILEX

[A. Toschi et al., PRB (2007); O. Gunnarsson et al., PRB (2016) T. Ayral et al., PRB (2016); G. Rohringer et al., PRB (2018); ….]



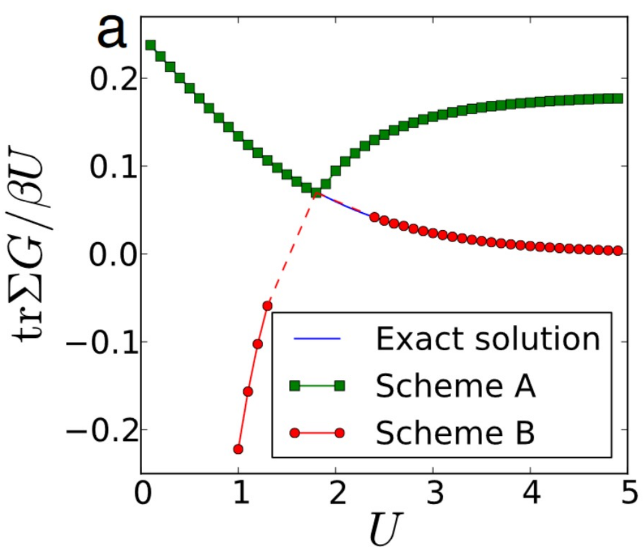
II. Multivaluedness of LW functional

iterative/self-consistent (=``bold'')
Diagrammatic resummation

Diagrammatic Monte Carlo

Nested Cluster Schemes

[E. Kozik et al., PRL (2015); A. Stan et al., NJP (2015); R. Rossi et al., PRB (2015); J. Vucicevic, et al. PRB (2018), ….]



The underlying physics of the nonperturbative regime

enhanced
local spin
fluctuations

STRONG INTERTWINING between CHANNELS

suppressed
local pair
fluctuations

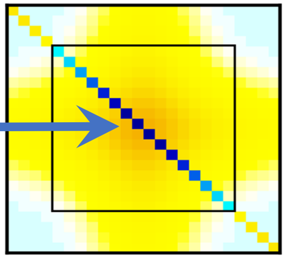
suppressed
local charge/density
fluctuations

- S.Adler…. & AT, SciPost (2024)
- T.Mazitov & A. Katanin, PRB (2022)

- **a coherent picture:**

local moment

suppression



of charge fluct.

2PI-vertex
divergences

Outline

Intro: challenging aspects of many-electron theory

1. DMFT as a starting point

2. Basics of its diagrammatic extensions

3. How ***nonperturbative*** information is ``encoded'' in the DMFT vertex functions

4. DΓA algorithms: different flavours

5. Pedagogical discussion of relevant results

Conclusions & Outlook

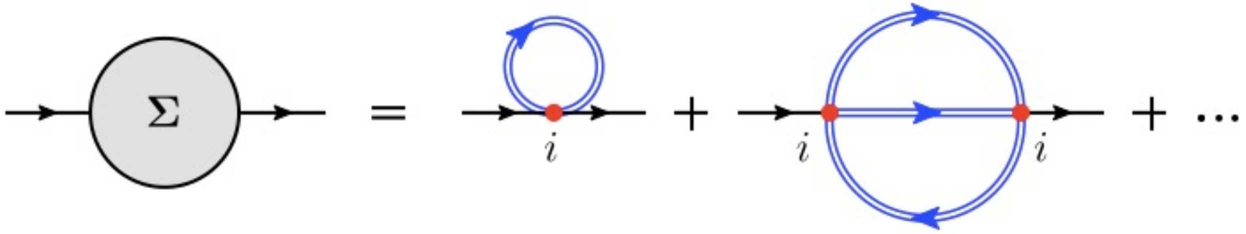
the dynamical vertex approximation (DΓA): a 2PI-based approach

AT, A. Katanin, K. Held, PRB (2007)

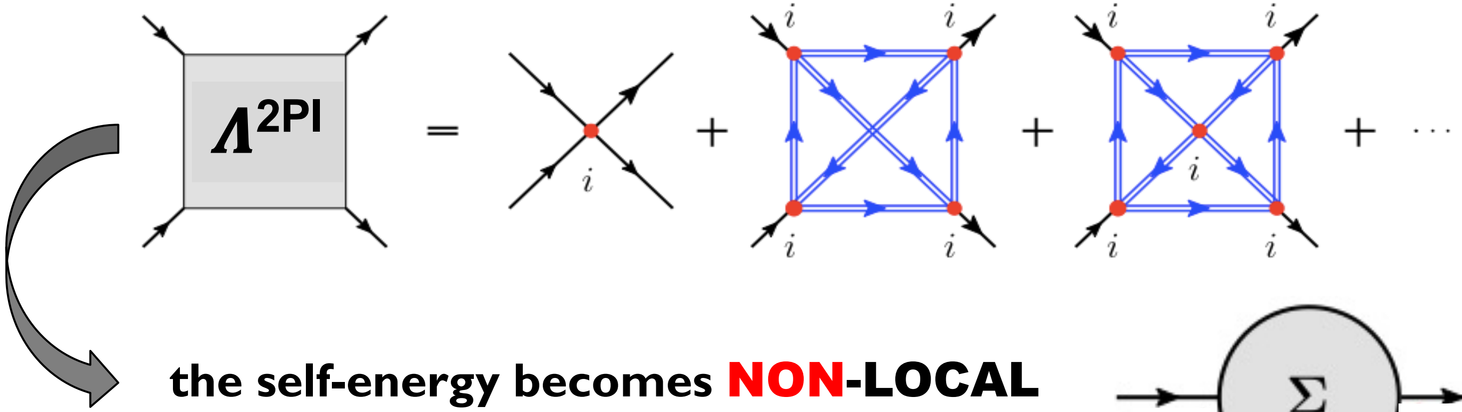
See also: PRB (2009), PRL (2010), PRL (2011), PRB (2012), PRB (2015), …

Review: RMP (2018)

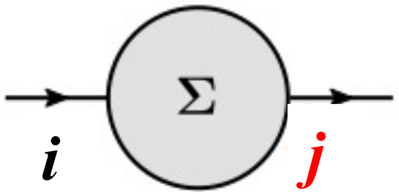
- **DMFT:** all **1-particle** irreducible diagrams (=self-energy) are **LOCAL !!**



- **DΓA:** all **2-particle** irreducible diagrams (=vertices) are **LOCAL !!**



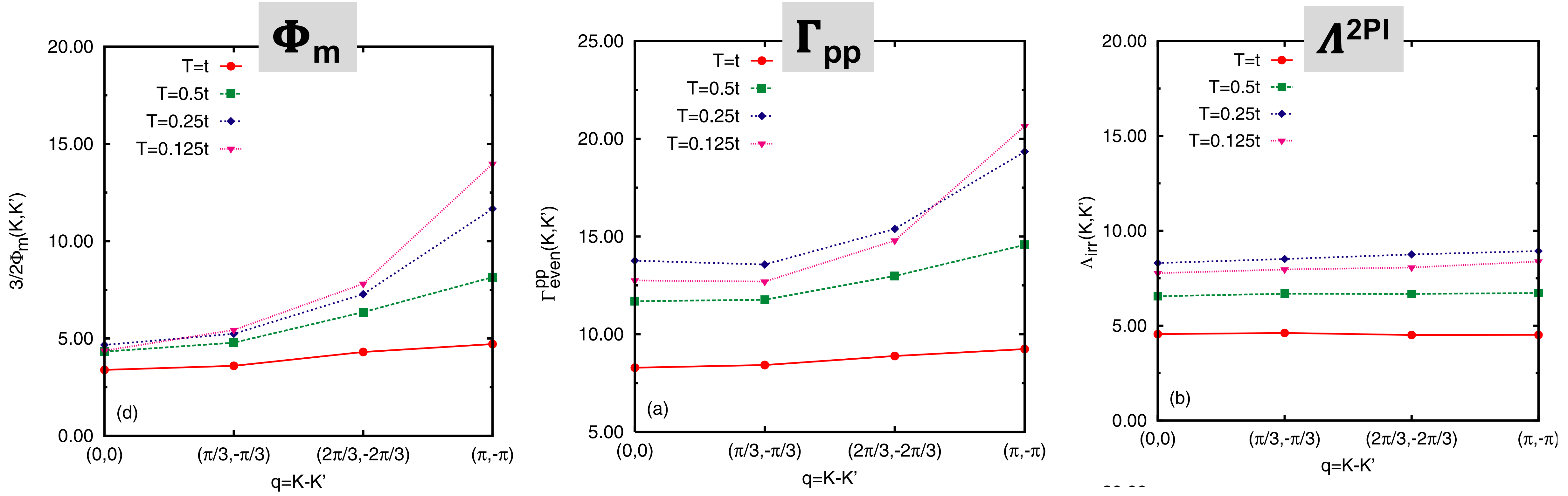
the self-energy becomes NON-LOCAL



An inspiring example : DCA calculations of **k**-dependent vertex functions

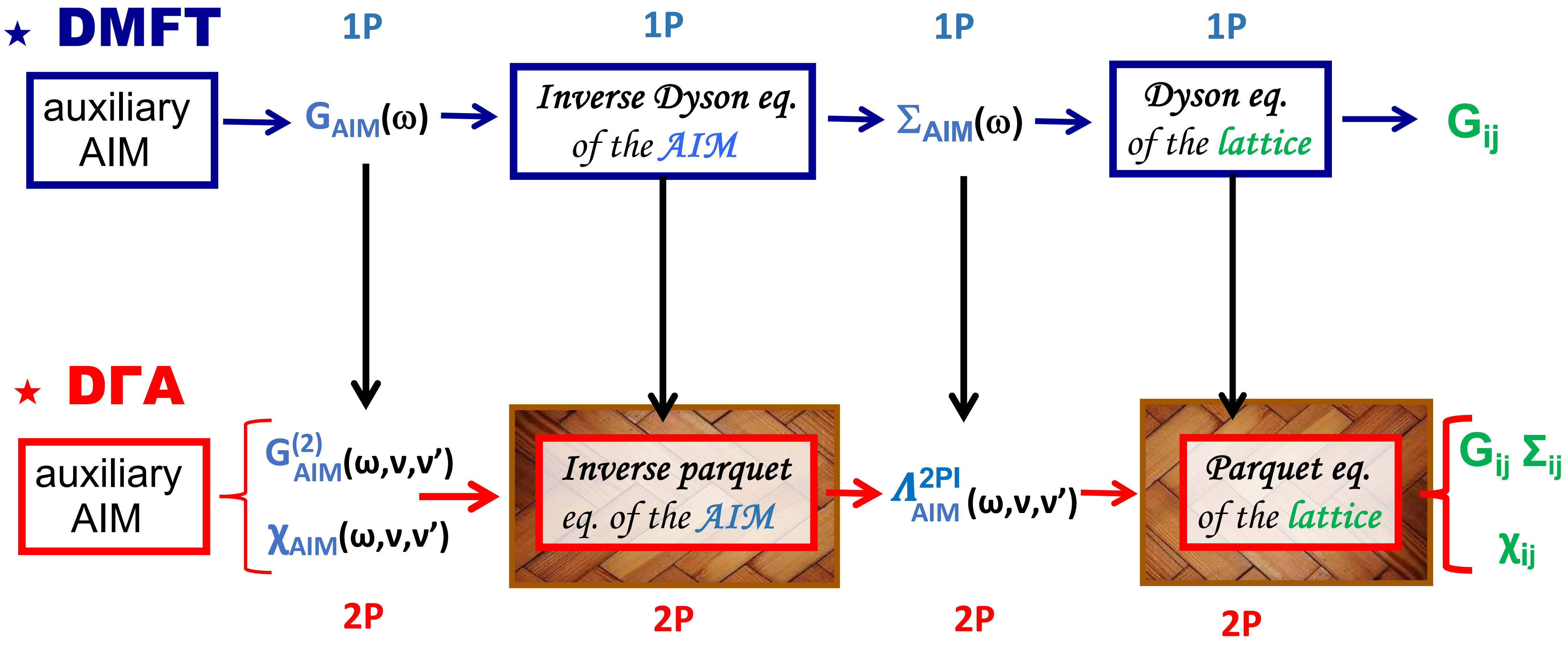
... of increasing 2P-irreducibility

2P-reducible → **2PI in pp channel** → **2PI in all channels**

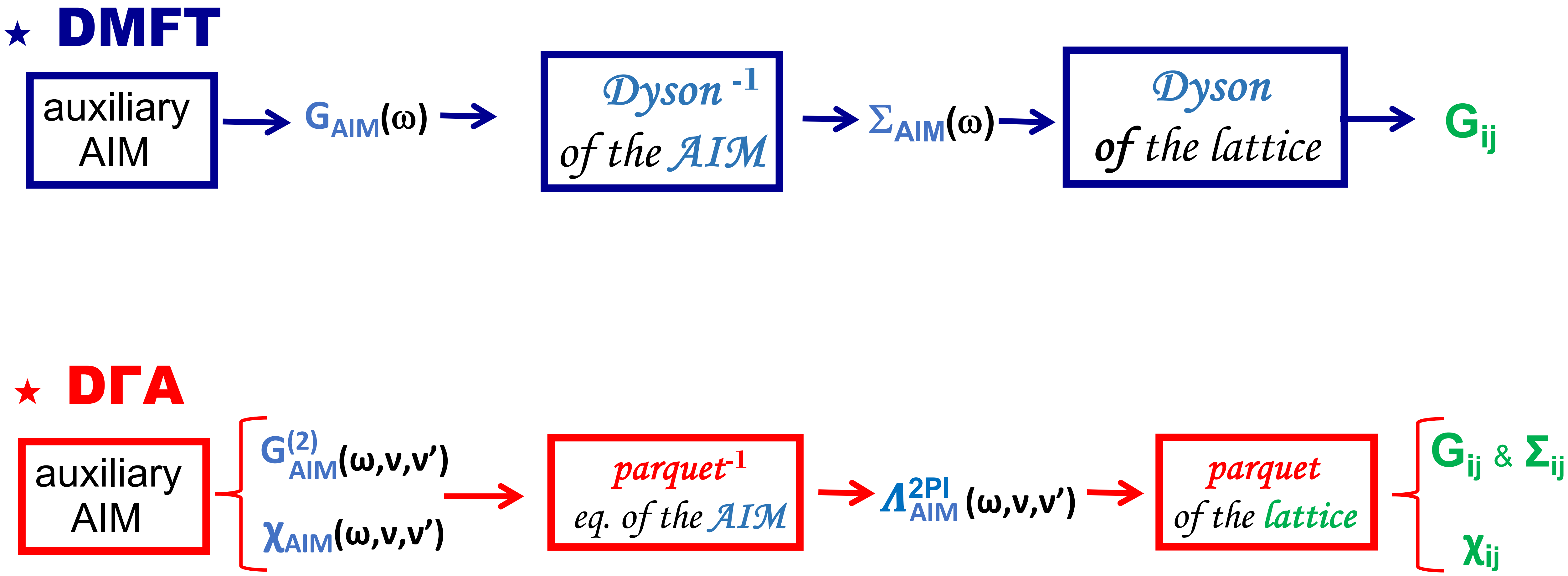


DCA, 2d-Hubbard model, **U**=4t, **n=0.85**, ν=ν'=π/β, ω=0, s. Th. Maier et al., PRL (2006)

Flowchart of the (full fledged) DΓA algorithm



Flowchart of the (full fledged) DΓA algorithm

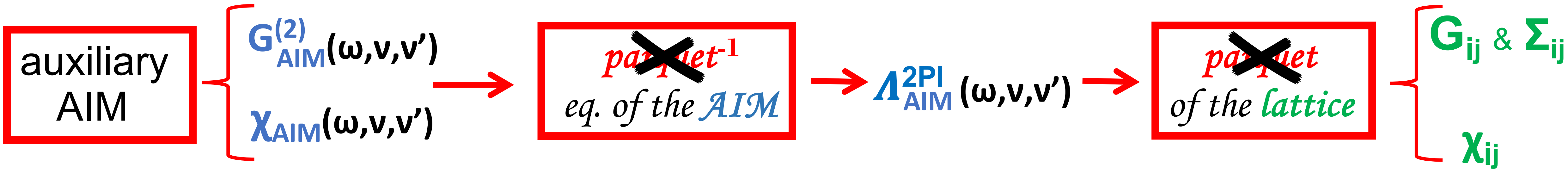


Flowchart of the full fledged DΓA algorithm

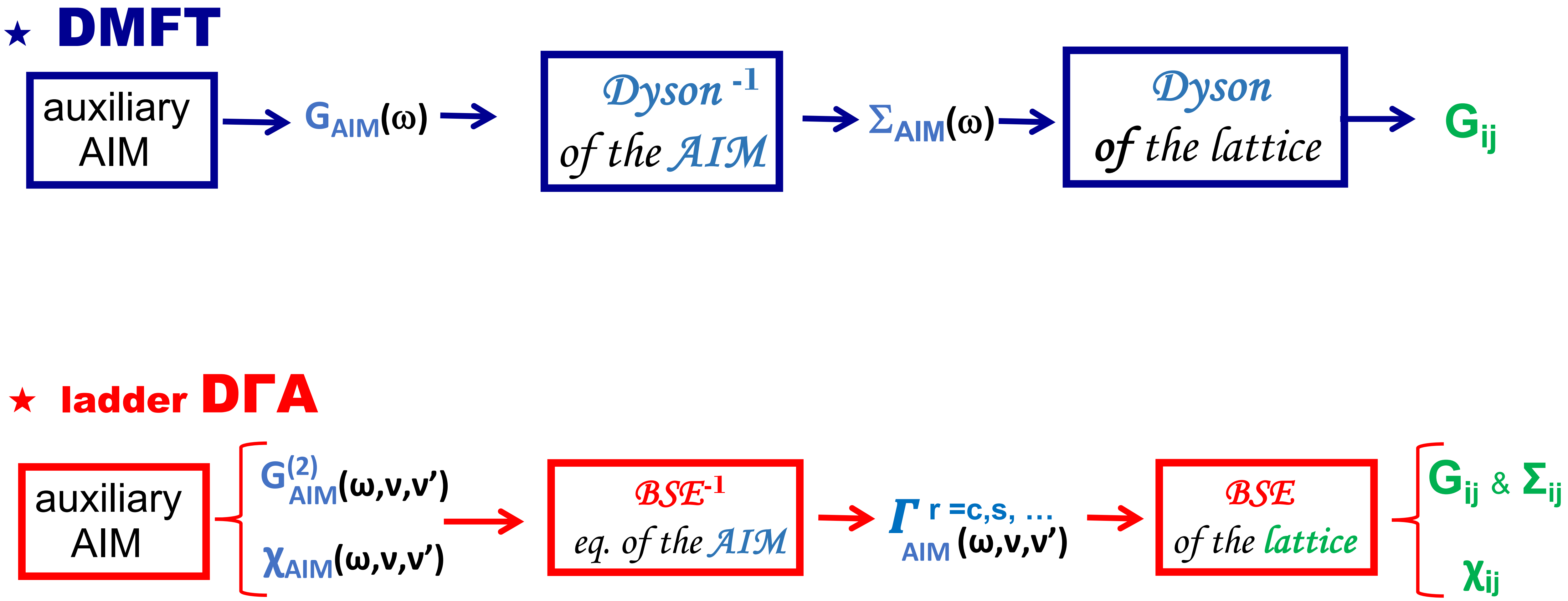
★ DMFT

→ *Possible simplification, if the **nonlocal fluctuation** of **one** channel dominate:*

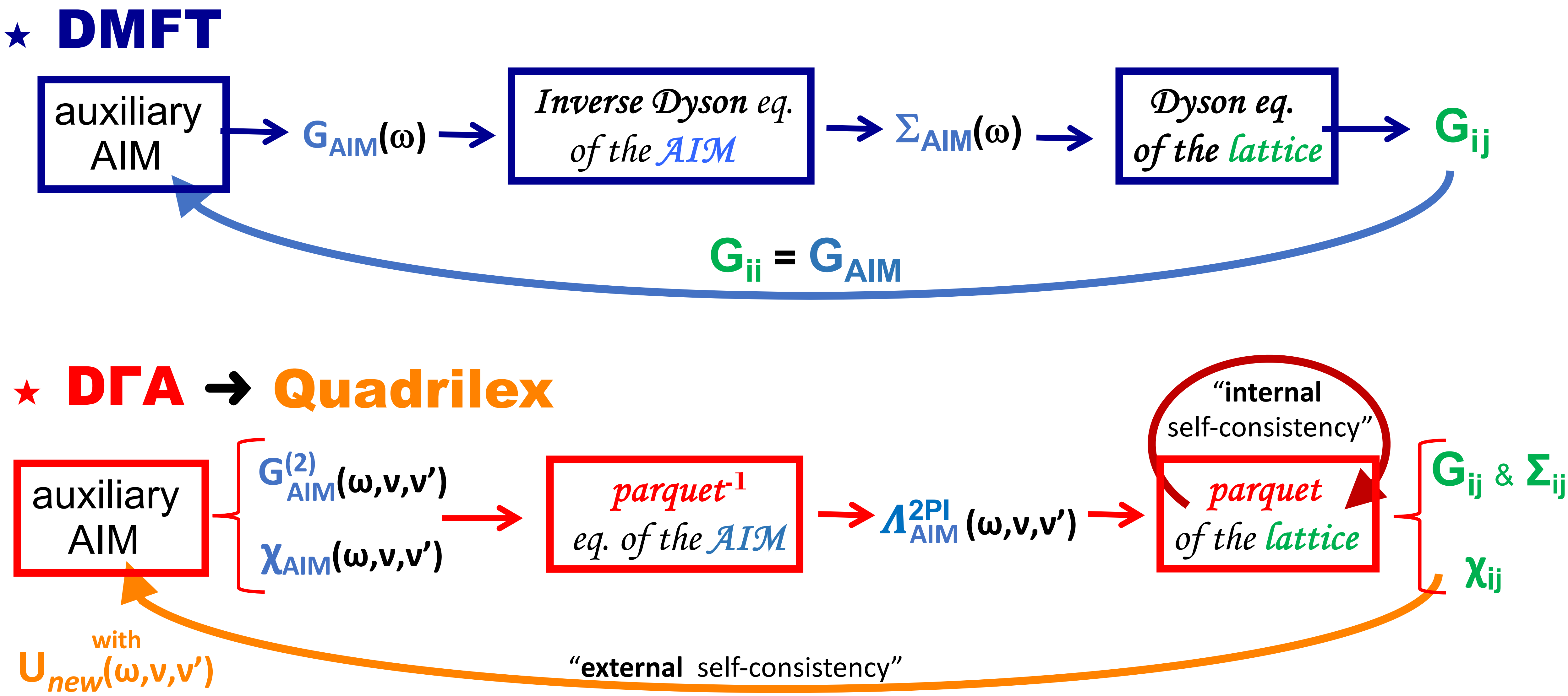
★ DΓA



*Possible simplification: the **ladder** DΓA algorithm*

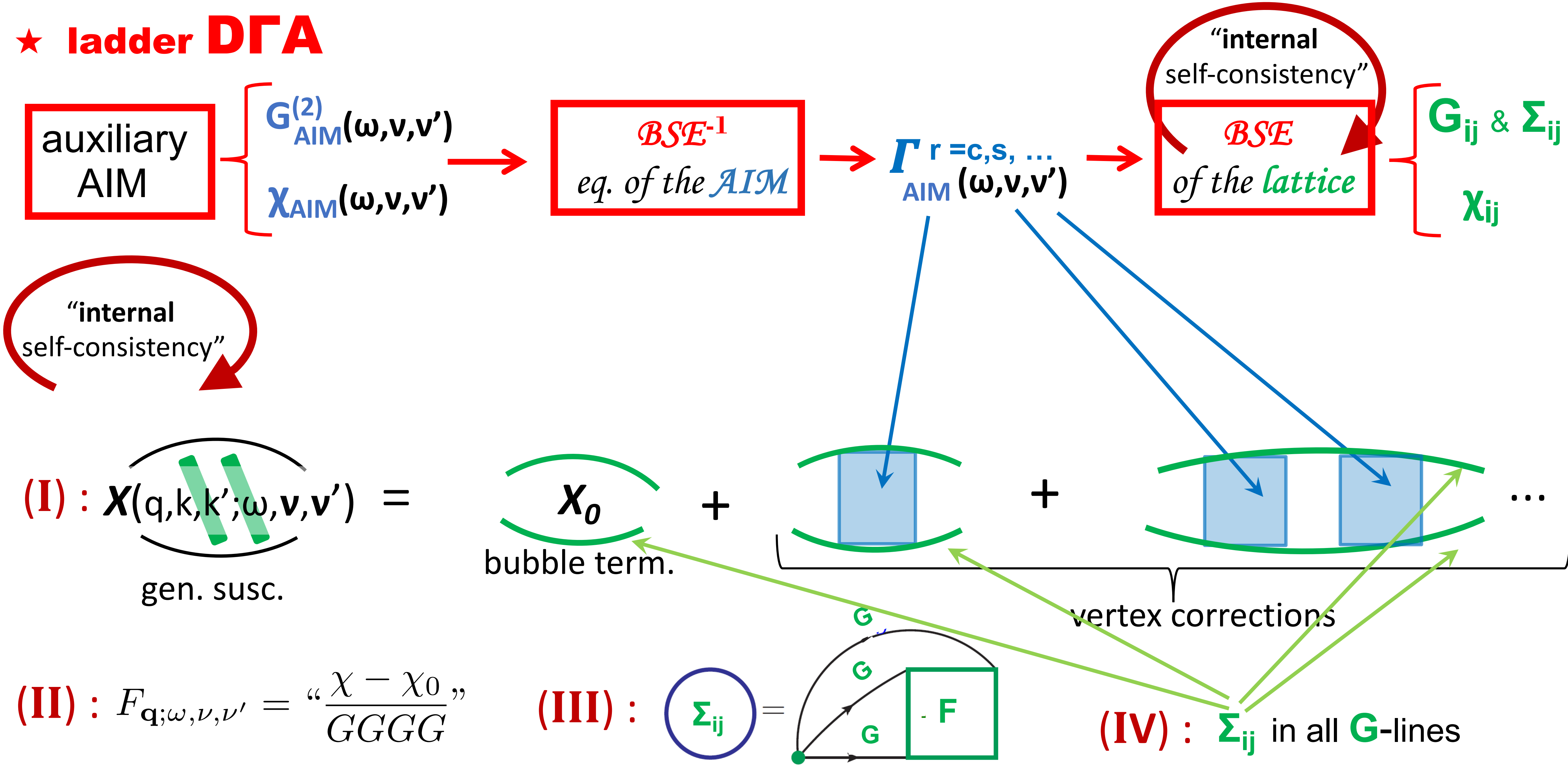


And … what about self-consistency?

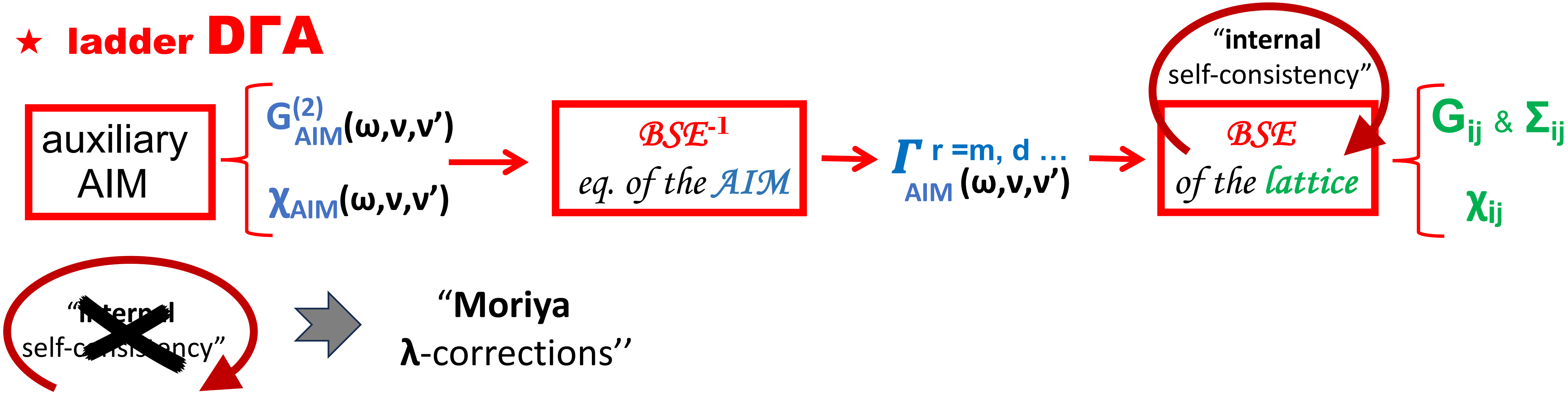


❖ For the theory of the **QUADRILEX** approach, s. T. Ayral & O. Parcollet, PRB (2016)

*Self-consistency of the **ladder** DΓA algorithm*



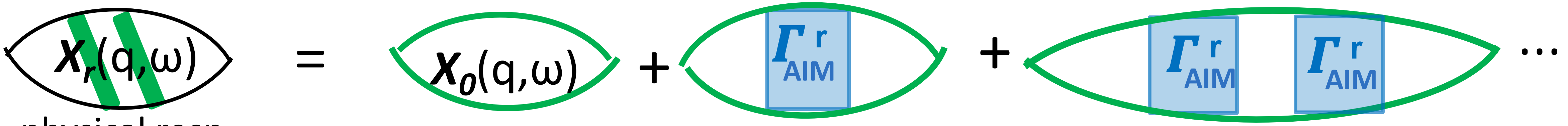
An approximation for the self-consistency: Moriya DΓA or ``λ-DΓA“



A. Katanin et al, PRB (2009); G. Rohringer & AT, PRB (2016); J. Stobbe & G. Rohringer, PRB (2023)

Explicit expressions for the Moriya DΓA or ``λ-DΓA“

(I) :



physical resp.
in channel *r (eg. m, d)*

bubble term.

(II) : $\chi^{\lambda}_{r=m,d,\cdots}(\mathbf{q},\omega) = \left([\chi_r(\mathbf{q},\omega)]^{-1} + \lambda_r\right)^{-1}$

Moriya "mass"-term correction **?**

To be determined
by enforcing TPSC/parquet-like sum rules

$$\frac{1}{2}\sum_{\omega\mathbf{q}}\left(\chi^{\lambda_d,\omega}_{d,\mathbf{q}} + \chi^{\lambda_m,\omega}_{m,\mathbf{q}}\right) \stackrel{!}{=} \frac{n}{2}\left(1-\frac{n}{2}\right),$$

$$\underbrace{\frac{U}{2}\sum_{\omega\mathbf{q}}\left(\chi^{\lambda_d,\omega}_{d,\mathbf{q}} - \chi^{\lambda_m,\omega}_{m,\mathbf{q}}\right) + U\frac{n^2}{4}}_{E^{(2)}_{\text{pot}}} \stackrel{!}{=} \underbrace{\sum_{\nu\mathbf{k}} G^{\boldsymbol{\lambda},\nu}_{\mathbf{k}}\Sigma^{\boldsymbol{\lambda},\nu}_{\mathbf{k}}}_{E^{(1)}_{\text{pot}}}$$

(III) : $$\Sigma^{\boldsymbol{\lambda},\nu}_{\mathbf{k}} = \frac{Un}{2} - U\sum_{\omega\mathbf{q}}\left[1 + \frac{1}{2}\gamma^{\nu\omega}_{d,\mathbf{q}}\left(1 - U\chi^{\lambda_d,\omega}_{d,\mathbf{q}}\right) - \frac{3}{2}\gamma^{\nu\omega}_{m,\mathbf{q}}\left(1 + U\chi^{\lambda_m,\omega}_{m,\mathbf{q}}\right) - \sum_{\nu'}\chi^{\nu'\omega}_{0,\mathbf{q}}F^{\nu\nu'\omega}_{m}\right]G^{\nu+\omega}_{\mathbf{k}+\mathbf{q}},$$

with $\gamma^{\nu\omega}_{r,\mathbf{q}} = \sum_{\nu'}\left(\chi^{\nu\nu'\omega}_{0,\mathbf{q}}\left(1 \pm U\chi^{\omega}_{r,\mathbf{q}}\right)\right)^{-1}\chi^{\nu\nu'\omega}_{r,\mathbf{q}}$ and $\chi^{\nu\nu'\omega}_{0,\mathbf{q}} = -\beta\delta_{\nu\nu'}\sum_{\mathbf{k}} G^{\nu}_{\mathbf{k}}G^{\nu+\omega}_{\mathbf{k}+\mathbf{q}}$, for further details, s. J. Stobbe & G. Rohringer, PRB (2023).

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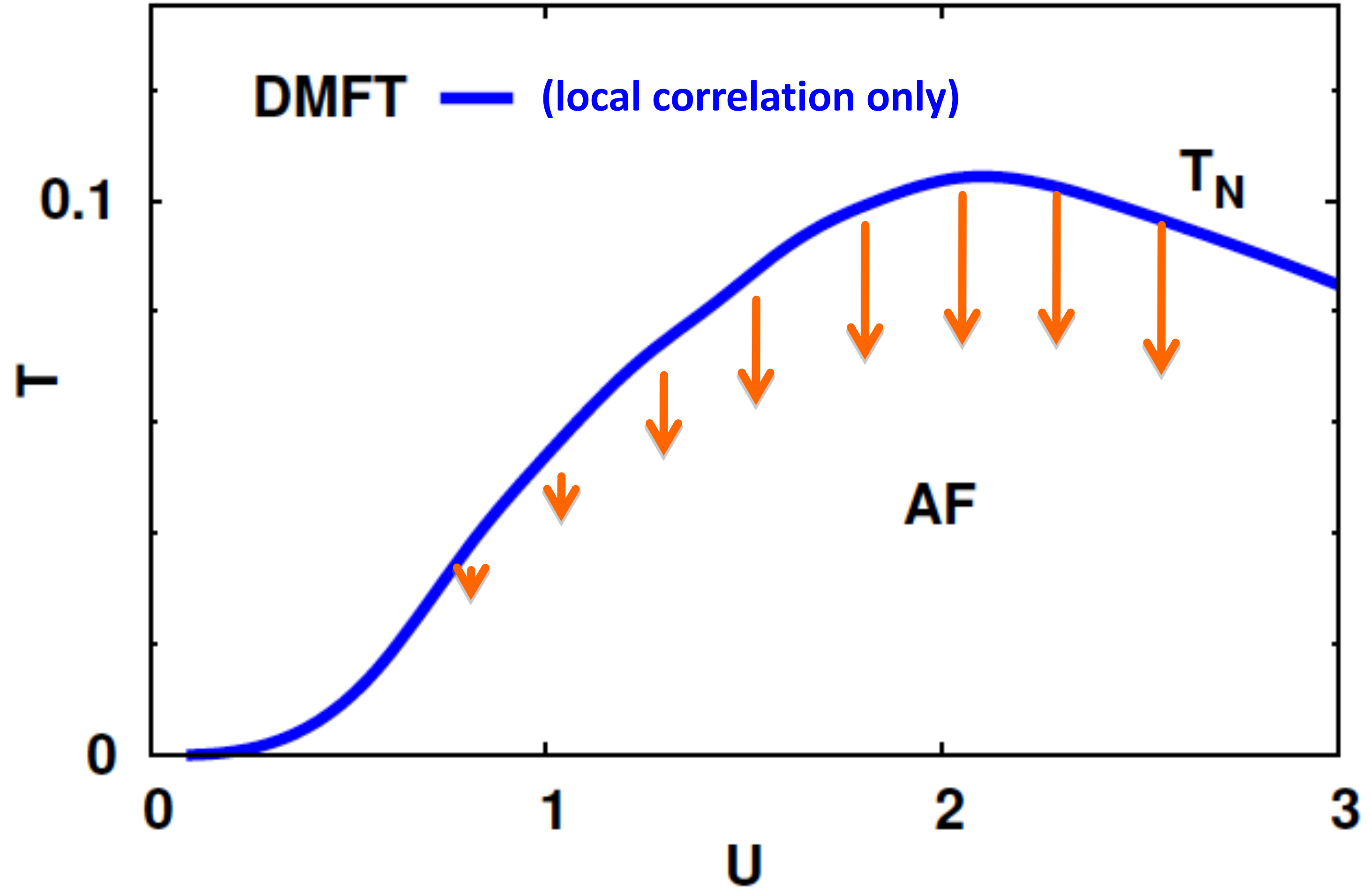
DΓA (=beyond DMFT) results in 3 dimensions

✔ **phase diagram** computed by **λ-DΓA:**

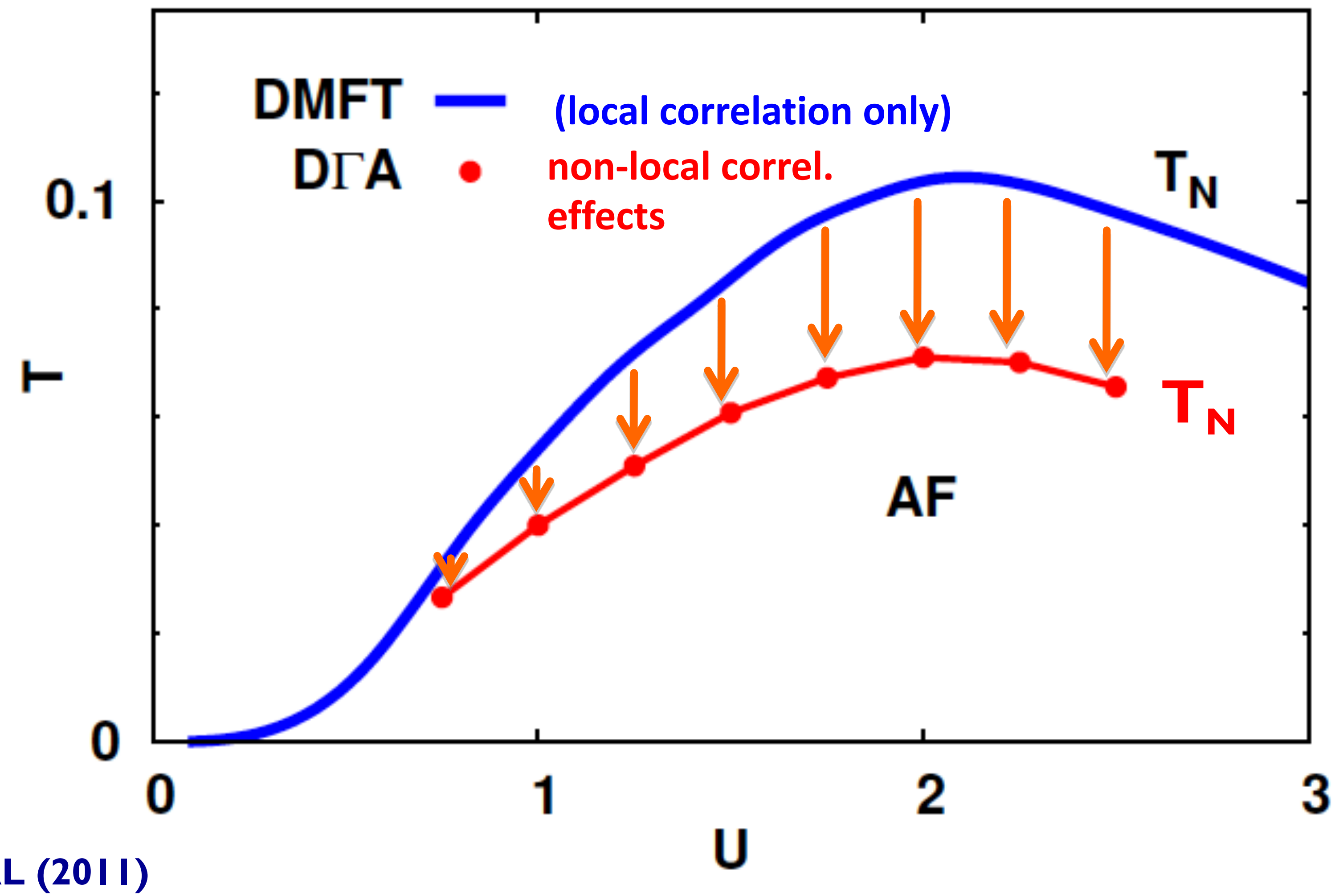
single band **Hubbard model** with nearest neibouring hopping in **d=3** (@ half-filling), in unit of $t = \frac{2}{\sqrt{6}}$



λ-DΓA results in 3 dimensions

✔ **phase diagram**: one-band Hubbard model in **d=3** (half-filling)

G. Rohringer, AT, et al., PRL (2011)

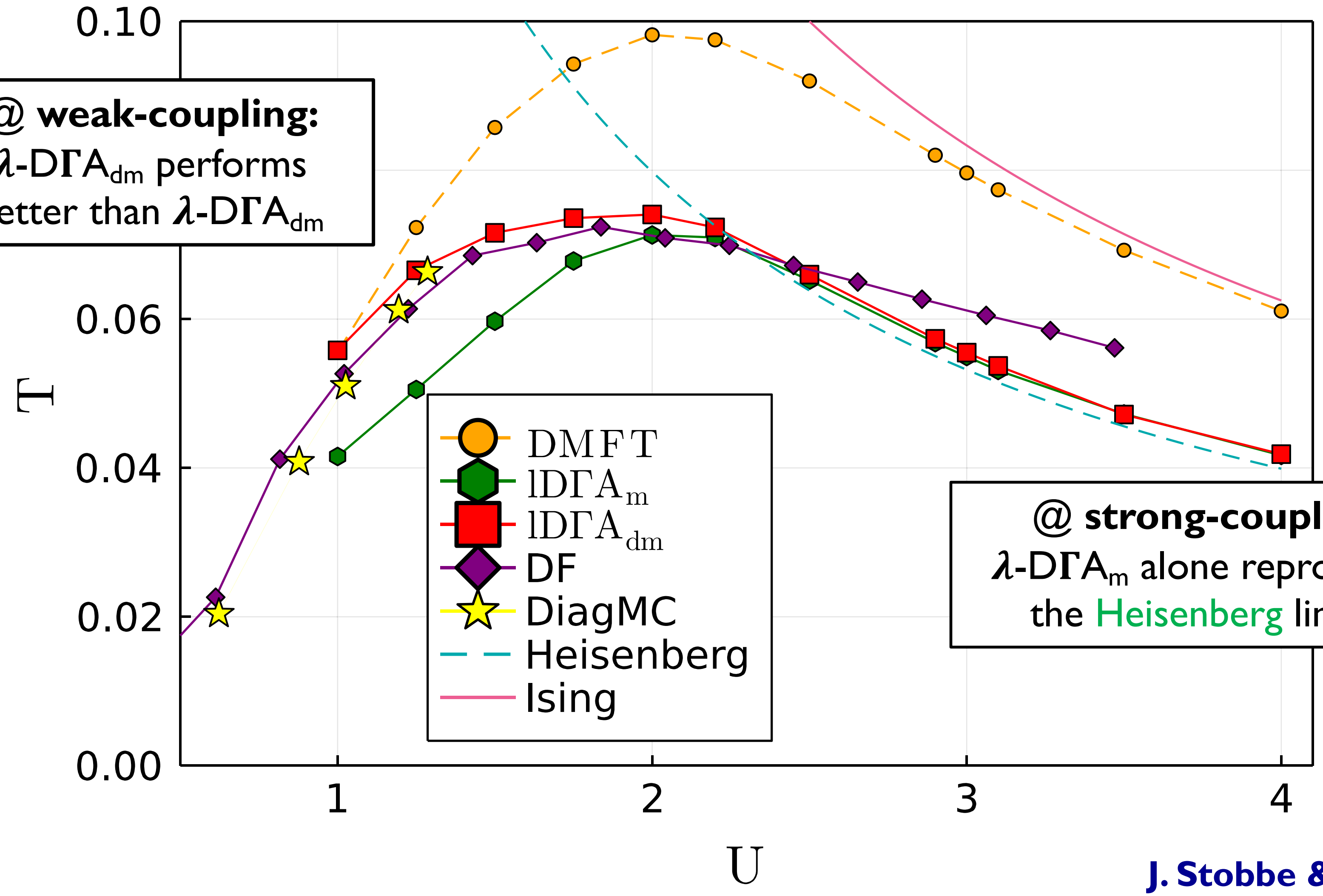


G. Rohringer, AT, et al., PRL (2011)

DΓA results in 3 dimensions: a quantitative comparison

✔ **phase diagram** computed by **λ-DΓA:**

single band **Hubbard model** with nearest neibouring hopping in **d=3** (@ half-filling)



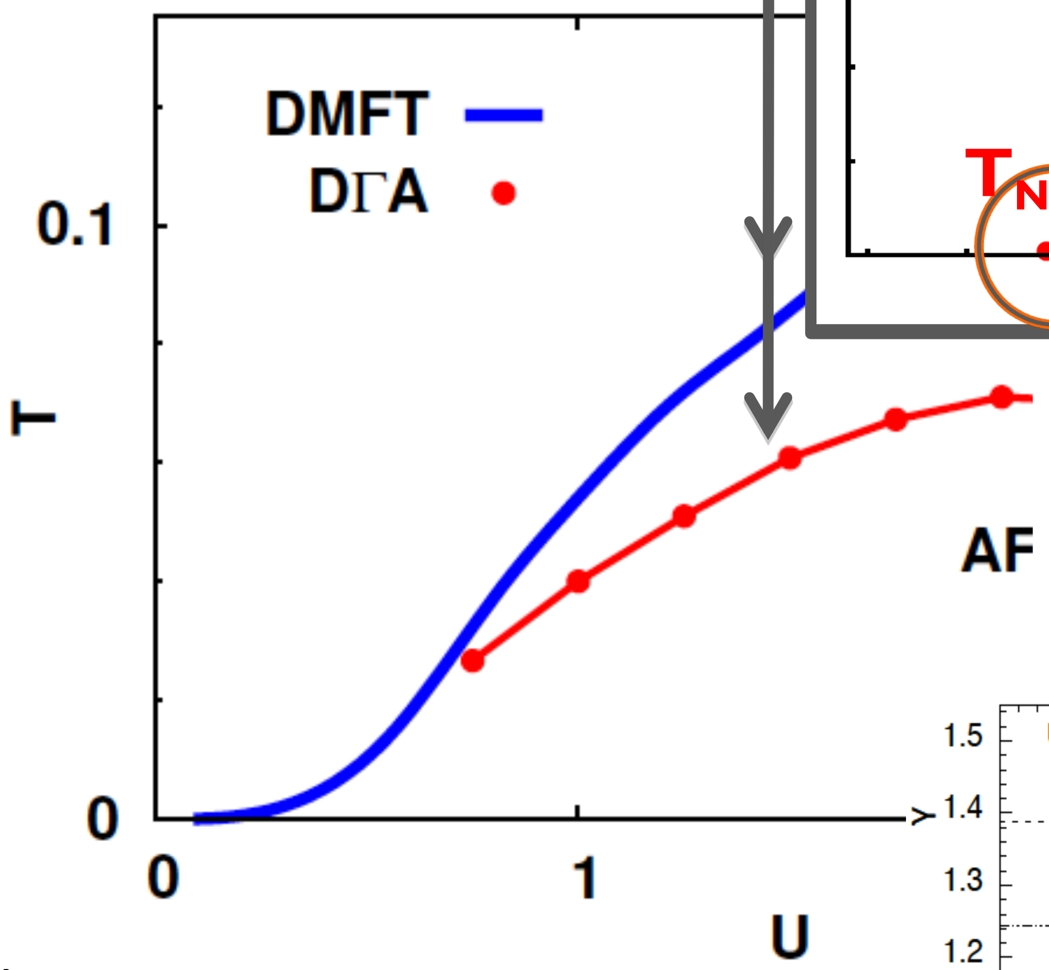
@ weak-coupling:
λ-DΓA$_{dm}$ performs better than λ-DΓA$_{dm}$

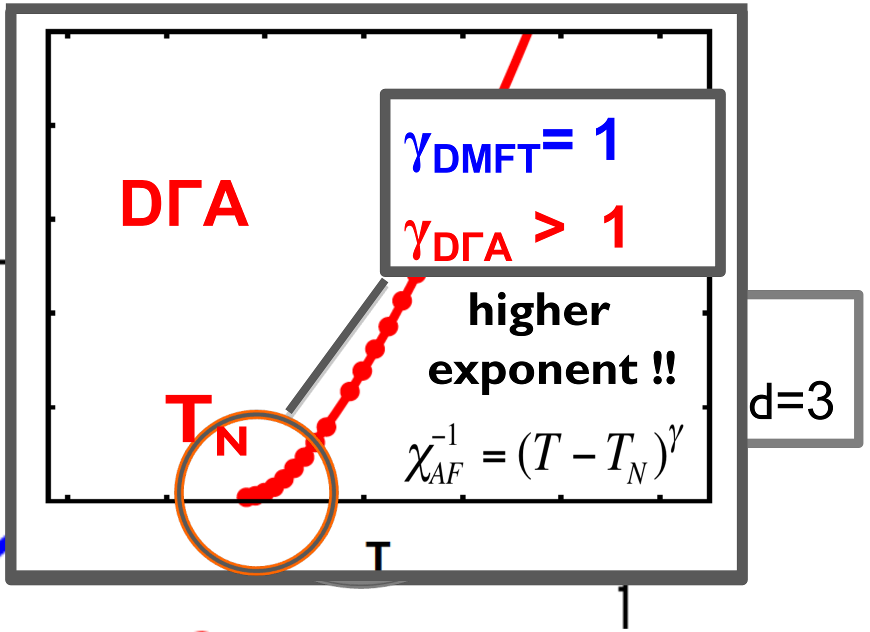
@ strong-coupling:
λ-DΓA$_m$ alone reproduces the Heisenberg limit !

J. Stobbe & G. Rohringer, PRB (2023)

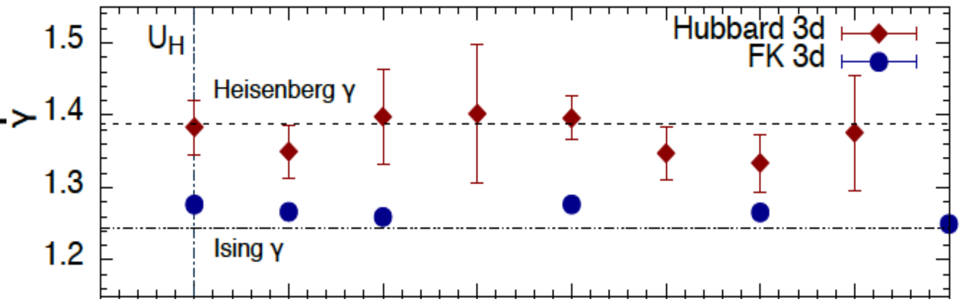
λ-DΓA results: the *critical* region

$$\chi_{AF}^{-1} = \chi_S^{-1}(q = (\pi,\pi,\pi))$$





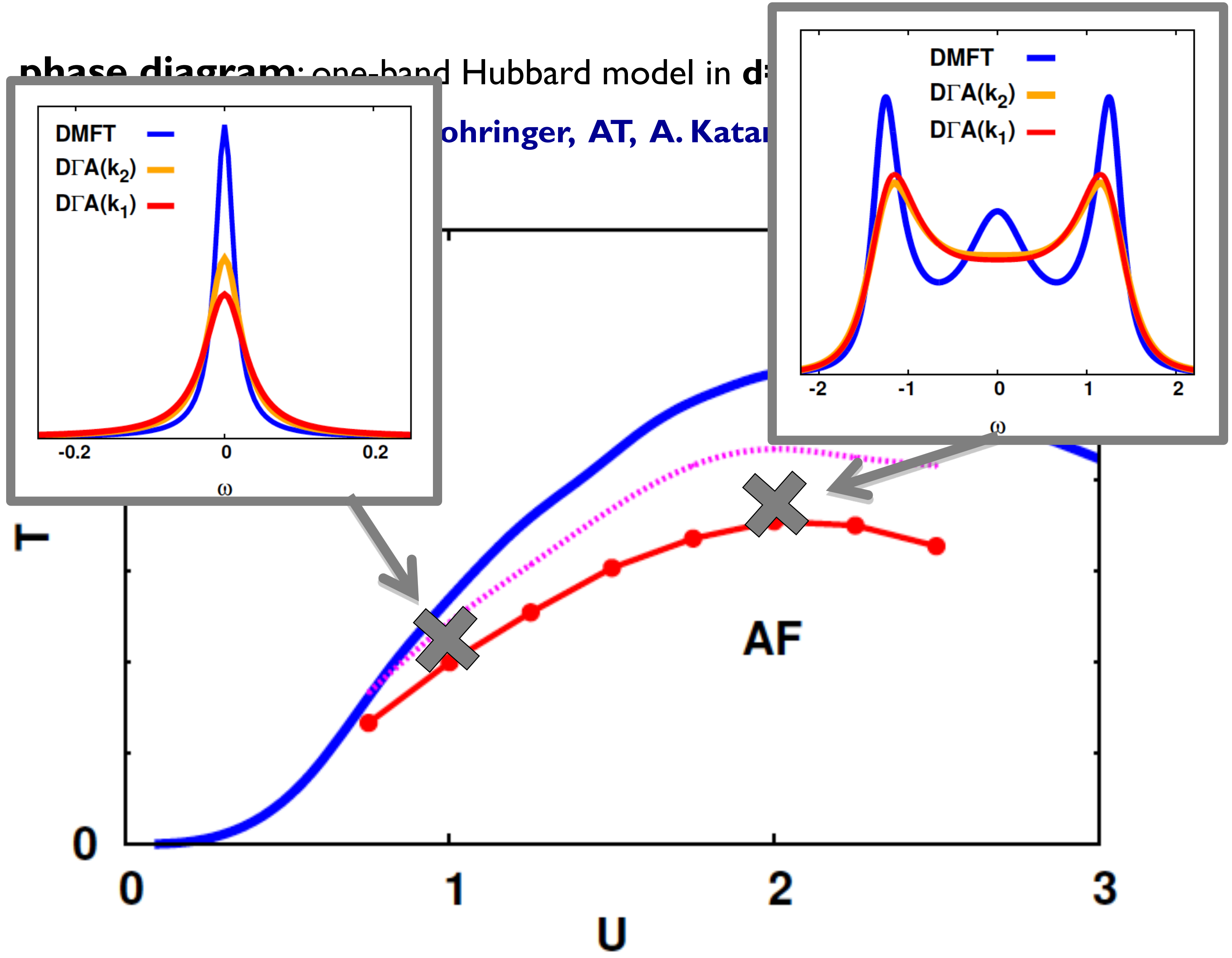
see also:
DF for Falicov-Kimball model
A. Antipov et al., PRL (2014)
and for the 3d-Hubbard
D. Hirschmeier et al., PRB (2015)



For the critical exponent of λ-DΓA see: **L. Del Re, … & AT, PRB (2019)**

λ-DΓA results in 3 dimensions: the spectral properties

✔ **phase diagram**: one-band Hubbard model in **d**

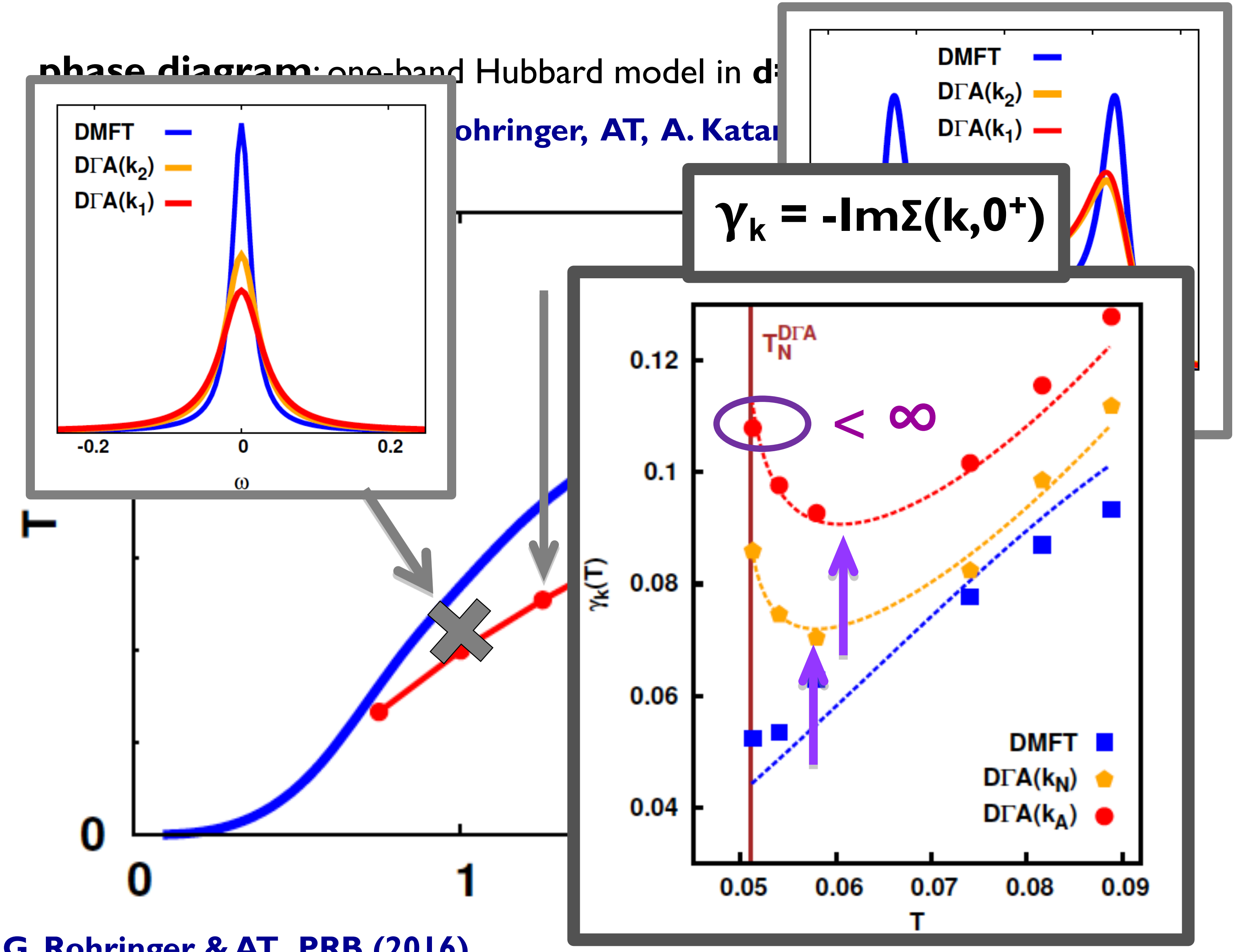


A. Katanin et al, PRB (2009); G. Rohringer & AT, PRB (2016)

λ-DΓA results in 3 dimensions: the spectral properties

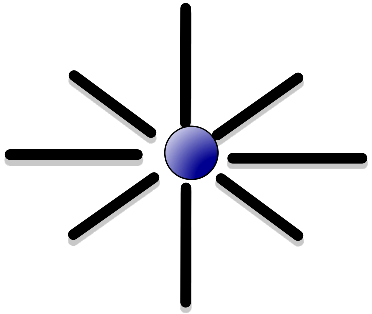
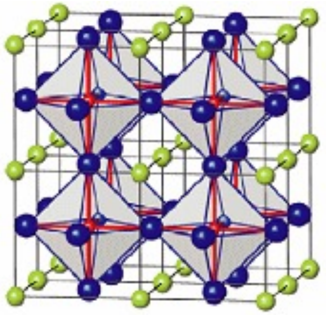
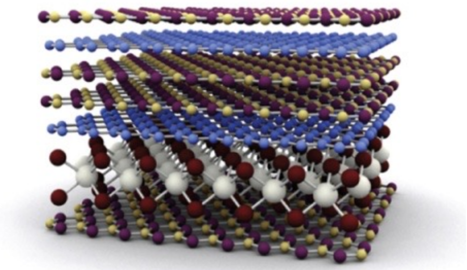
✔ **phase diagram**: one-band Hubbard model in **d**=

ohringer, AT, A. Katar

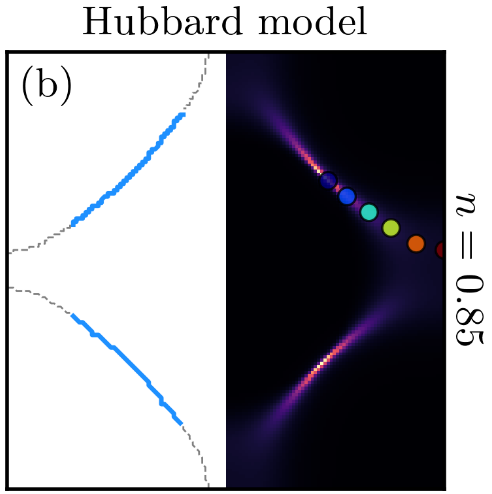


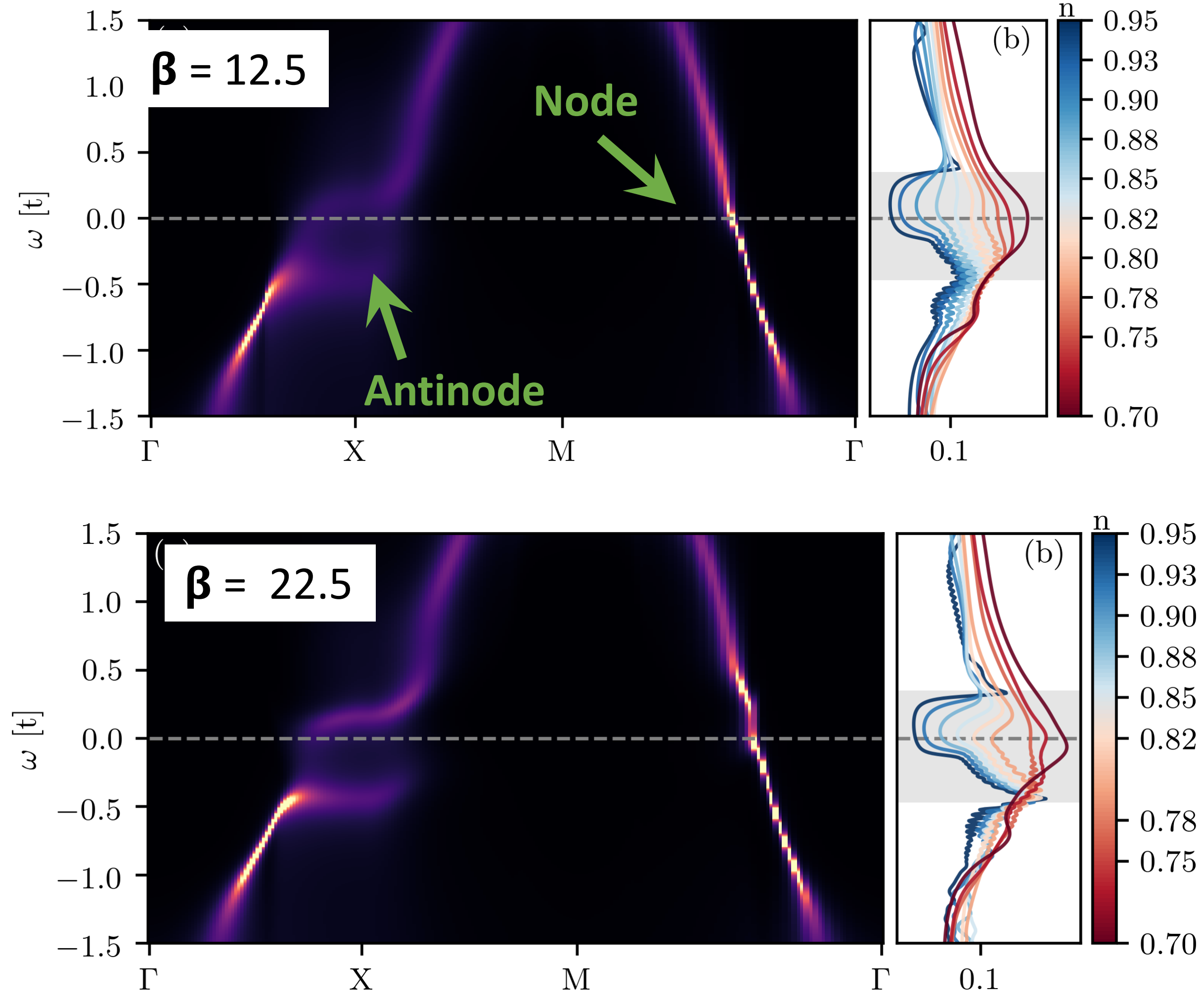
G. Rohringer & AT, PRB (2016)

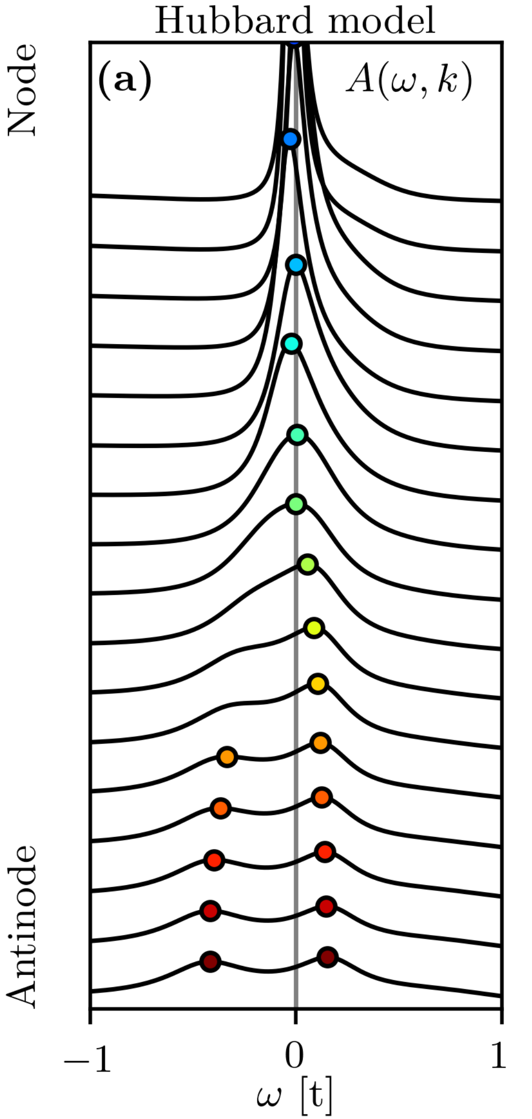
From ∞ dimensions to … „reality“ !

Dimensionality:	Systems:		Spatial correlations **beyond** DMFT:
d = ∞		mathematical **idealization** (DMFT exact!)	None
d = 3		**bulk materials**	in some conditions
d = 2		layered compounds: 2D-networks, adatoms **heterostructures**	quite strong!!

„There is life“ … out of half-filling: *λ-DΓA study of the* pseudogap in 2D



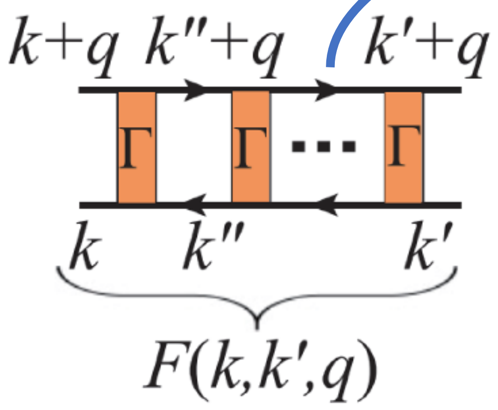


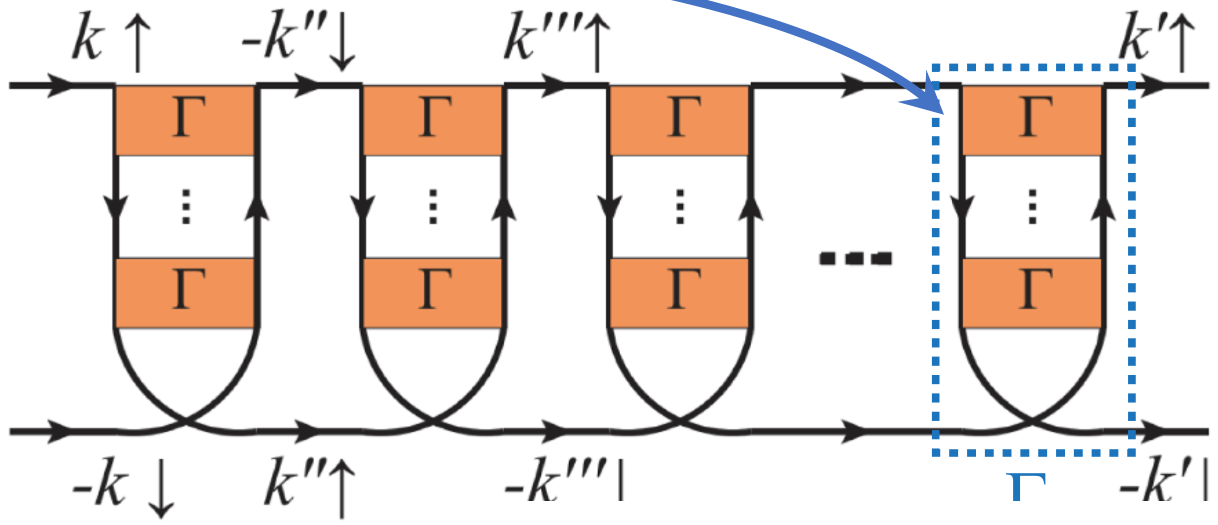


2D-Hubbard model, **U=8t (t'=-0.2 t; t''=0.1t)**, s. **P. Worm, M. Reitner … & AT arXiv:2312.xxxx**

„There is life“ out of half-filling: d-wave superconducting instabilities in λ-DΓA

λ-DΓA in the magnetic channel

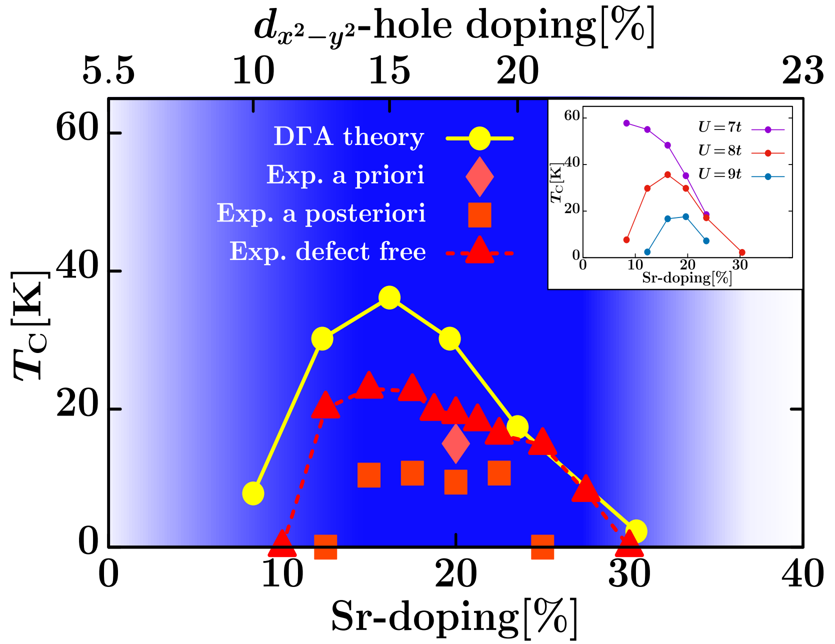




linearized instability (Eliashberg) equation:

$$\lambda\Delta(k) = -\frac{1}{\beta N_k}\sum_{k'}\Gamma_{\mathrm{pp}}(k,k',q=0)G(k')G(-k')\Delta(k')$$

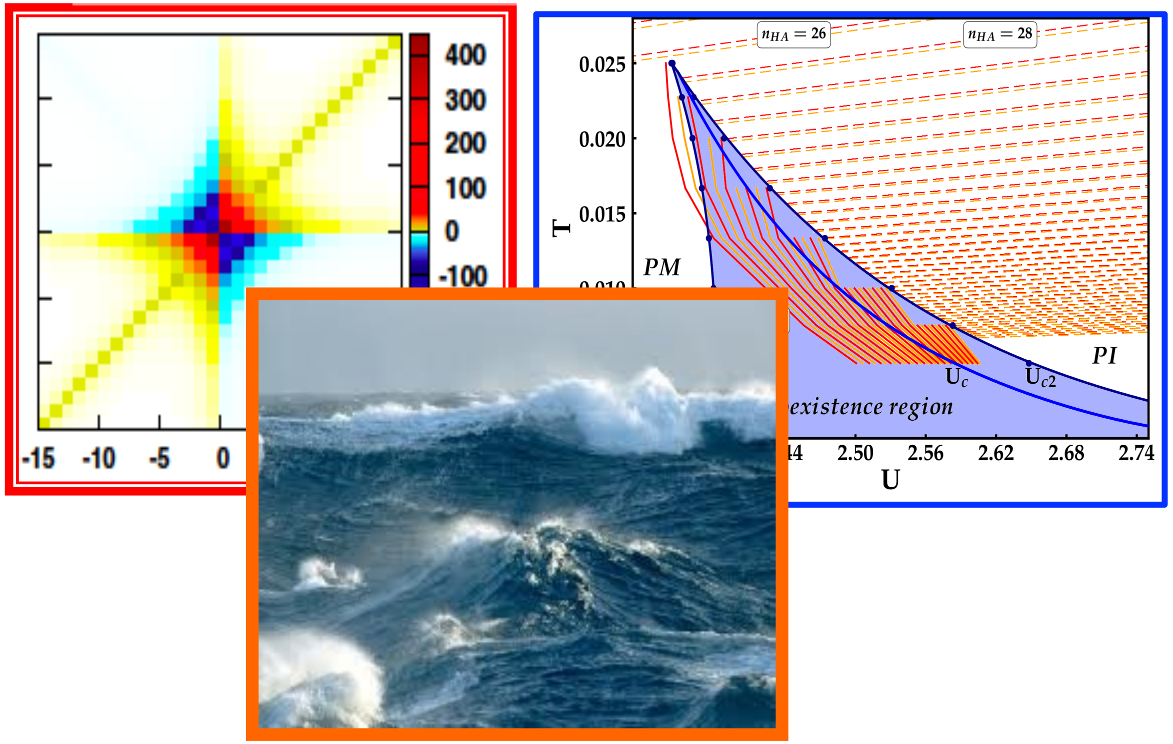
λ-DΓA for $Sr_{0.2}Nd_{0.8}NiO_2$



❖ M. Kitatani et al., PRB (2019) & J. Cond. Mat. (2022)

A (diverging) elephant in the room (of DΓA) ?

$\Gamma^r = \infty$



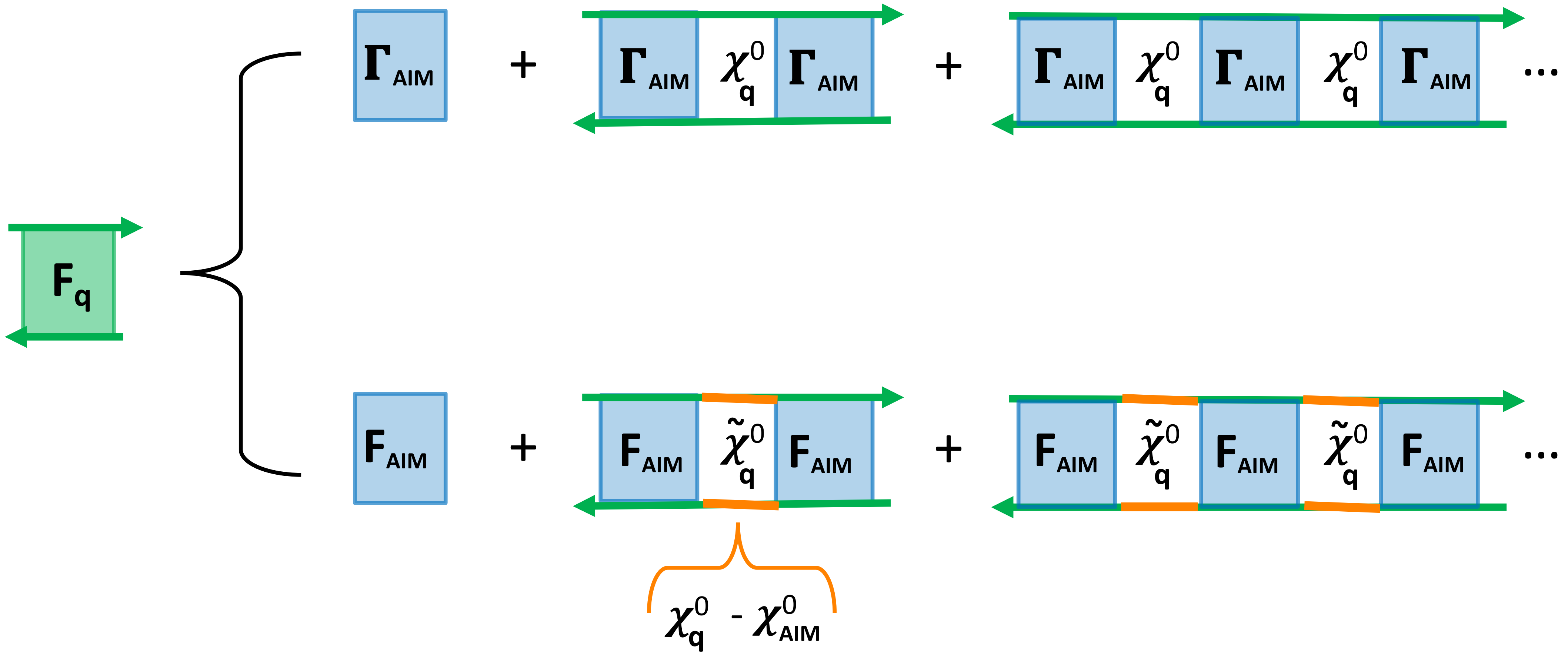
Should be DΓA called rather ...``D∞A“ ?

[credit: G. Sangiovanni, 2013]

*No, luckly this elephant... is **not** a dangerous one:*

Working with 2PI vertices : The (non-problematic) ladder case

By exploiting the following identity the ladder diagrams of DΓA can be **exactly** rewritten …



… eliminating any explicit appearance of the (possibly dangerous) Γ_{AIM} !

G. Rohringer, AT , et al., PRB (2013); G. Rohringer et al., RMP (2018)

And, remarkably, this works **even** *at the parquet-DΓA level !!*

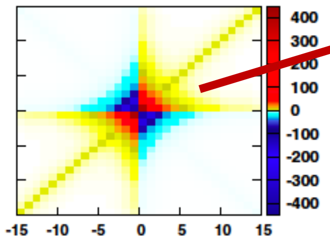
✓ Important progress achieved by *Jae-Mo Lihm*, *Seung-Sup Lee*, *F. Kugler* & *D. Kiese*

(no spoiler here: preprint in preparation)

Conclusions:

Diagrammatic extenstions of DMFT: **two**-step procedure !

\# **step 1**: extract a **local vertex** from DMFT/EDMFT(AIM)



\# **step 2**: build upon that the **diagrammatic** expansion (e.g., 2.order, ladder, parquet, …)

Inclusion of the **nonperurbative** **local** physics: local moments

Mott MIT, etc.

☆ ***DΓA*** input: **2PI**-vertex functions of DMFT

- full fledged: parquet resummation
- approximated: ladder resummation
  - self-consistent
  - Moriya corrections

Outlook: ***Multiscale approaches***

→ vertices of C-DMFT/DCA (short range correlations) input of **diagrammatic expansions** (long range ones)